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ION-CYCLOTRON TURBULENCE AND DIAGONAL DOUBLE LAYERS IN A MAGNE-TOSPHERIC PLASMA

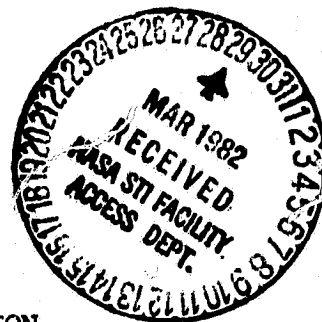V. A. Liperovskiy, M. I. Pudovkin, G. A. Skuridin, S. L. Shalimov

STANDARD TITLE PAGE

1. Report No. NASA TM-76742	2. Government Accession No.	3. Recipient's Catalog No.

4. Title and Subtitle ION-CYCLOTRON TURBULENCE AND DIAGONAL DOUBLE LAYERS IN A MAGNETOSPHERIC PLASMA

5. Report Date OCTOBER 1981

6. Performing Organization Code

7. Author(s) V. A. Liperovskiy, M. I. Pudovkin, G. A. Skuridin, S. L. Shalimov

8. Performing Organization Report No.

10. Work Unit No.

9. Performing Organization Name and Address
SCITRAN
Box 5456
Santa Barbara, CA 93108

11. Contract or Grant No. NASw 3542

12. Type of Report and Period Covered
Translation

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, D.C. 20546

14. Sponsoring Agency Code

16. Abstract

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It examines the question of anomalous resistance governed by electrostatic ion-cyclotron turbulence, and one-dimensional and two-dimensional models of double electrostatic layers in the magnetospheric plasma.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	UNLIMITED - UNCLASSIFIED

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 63	22. Price

ANNOTATION

This work provides an analytical survey of current concepts regarding electrostatic ion-cyclotron turbulence (theory and experiment), and regarding inclined double potential layers in the magnetospheric plasma.

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ION-CYCLOTRON TURBULENCE AND OBLIQUE DOUBLE LAYER IN MAGNETOSPHERIC PLASMA

By

V. A. Liperovskiy, M. I. Pudovkin, G. A. Skuridin, and S. L. Shalimov*

This survey reflects the rising interest in studying electrostatic ion-cyclo- /3**
tron (EIC) turbulence which developed because of a number of new measurements made in the near-earth space plasma on the S3 - 3 satellite.

In addition to experimental proof of the EIC-waves in the magnetospheric plasma, the survey examines the main conclusions of the linear theory of EIC-instability. A detailed study of it is found in the pioneer works of Drummond and Rosenbluth [1], and Lominadze and Stepanov [2].

Study of the nonlinear stage of EIC-instability evolution was made by Petviashvili [3], Dum and Dupree [4] and others. Sagdeyev [6, 7, 8], Sagdeyev and Galeyev [9], as well as the book of Artsimovich and Sagdeyev [10] examined the problem of anoma- lous resistance governed by plasma turbulence. Kindel [11], Ionson [12], Ionson et al. [13] Dakin et al. [14] and Hudson et al. [15] examined a similar problem in relation to EIC-turbulence.

The appropriate experimental laboratory studies of EIC-turbulence were made. The most important has become the experiments of Dakin et al. [14], Correll et al. [16,17], and Bohmer and Fornaca [18]. As applied to longitudinal currents in the auroral zone of the magnetosphere, the role of the EIC – instability was first established by Kindel and Kennel [19], who /4 found that, among the basic three wave instabilities excited in a plasma with a current (Buneman, ion-sound, and EIC), the EIC-instability has a lower excitation threshold in the magnetosphere.

In 1963 Petviashvili showed that within the framework of one-dimensional quasilinear theory , in an infinite, uniform, magnetoactive plasma the occurrence of a plateau on the electron distribution function stabilizes the EIC-waves [3]. The effective frequency of electron collision with waves thus equals

$$\nu_* \simeq \nu_{ie} \left(1 + \frac{u}{v_e} \right)$$

where ν_{ie} are ion-electron collisions; u - drift velocity of electrons along the magnetic field which, for EIC-instability, has a much smaller thermal v_e. Such a small effect of collective collisions must naturally be disregarded.

Thus for a long time it was assumed that EIC-instability, in spite of a lower excitation threshold as compared with the ion-sound one and in spite of the absence of the requirement for non-isothermality, was not important in the problem of anomalous resistance in the magnetosphere.

It was confirmed by other authors – Vedenov [20], Lysak [22], Dum and Dupree [4] that collisions, three-dimensional effects and the presence of longitudinal constants of the electric fields can overcome the requirement for a"plateau", and saturation of the turbulence level is performed at a higher level (for example, under the influence of the mechanism of expansion of ion resonances [4]). As a result, the anomalous resistance caused by the EIC-turbulence increases. Experiments on the satellite S 3-3

[15] also showed that a "plateau" is not formed. Later in space experi-
ments it was found that the level of EIC-turbulence, connected with
longitudinal currents was so great that the frequency of collective
collisions of particles with waves leads to a large anomalous resistance
in the longitudinal currents. Thus the study of the anomalous resistance,
caused by EIC turbulence, is important for understanding many large-scale
geophysical processes.

The study [50] presents data on laboratory experiments indicating that
the development of strong wave turbulence of the ion-sound type leads to
the occurrence of direct potential layers in the current-carrying plasma,
i.e., layers in which the field is parallel to the magnetic field. The
double layers, which develop in the plasma at a very high EIC-turbulence
level, are inclined with respect to the magnetic field (so-called oblique
layers of the IC-type). The physical model of this inclined layer was
developed in a series of articles by Swift [51-55].

The basic features of this model are discussed below.

1. EIC-INSTABILITY: EXPERIMENT AND THEORY

1.1. EXPERIMENTAL DISCUSSION OF EIC-WAVES IN THE MAGNETOSPHERE

There have been very few direct observations of EIC-waves in the
magnetosphere up until recently.

EIC waves were observed in the region of diurnal polar cusps on the
satellite OGO-5 [42] during a strong magnetic storm on 1 November 1968.
A frequency was observed which corresponded to the intensity maximum of
waves located between 0.67 and 0.87 ω_{Bi}.

In a rocket experiment [25] at an altitude of about 600 km above the
Earth close to the auroral arc boundary strong electrostatic fluctuations
were determined about ω_{Bi} (at frequencies of 40 Hz). The

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simultaneously observed strong heating of the ions is associated by the authors of [25] with the presence of EIC instability in these regions.

Further, the electrostatic wave turbulence was studied in detail in [34,45], where a fairly high level of wave energy was noted at frequencies close to ω_{Bi}. These experiments, however, did not make a special study of the EIC waves. American works in the last several years from 1975-1978 which cover EIC turbulence in the cosmic plasma are studied in [43].

An important step forward towards studying wave turbulence in the magnetosphere was the studies of the Mozer group which were made based on measurements of the S 3-3 satellite [15,24]. These publications give the results of measurements (at altitudes about R_E in the auroral zone) of intensity of both the constant and variable δn of the plasma. The measurements showed the presence of regions of strong longitudinal electrical fields and wave electrostatic turbulence. These regions were recorded at varying local time (LT) on L-shells from L = 4.7 to L > 6. Typical dimensions of the regions were about 35 kilometers. This corresponds to 10 km in the ionosphere.

Figure 1.1 [24] presents an example of the measurements of spectral powers $(\delta n/n)^2$ and $(\delta E)^2$ for the 217th day of 1976. It should be noted that, in addition to these main measurements, measurements were made (evidently with much less accuracy) of the magnetic field. They made it possible to establish a rough estimate of the magnitude of the density of the longitudinal currents.

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Spectral Power [V²/(m < Hz)]

2,5×10⁰

2,5×10⁻¹

2,5×10⁻²

2,5×10⁻³

2,5×10⁻⁴

0   250   500

Frequency Hz
0121115 UT   Day 217, 1976

Spectral Power (< δn/n >2/Hz)

9,2×10⁻²

9,2×10⁻³

9,2×10⁻²

9,2×10⁻²

9,2×10⁻²

0   250   500

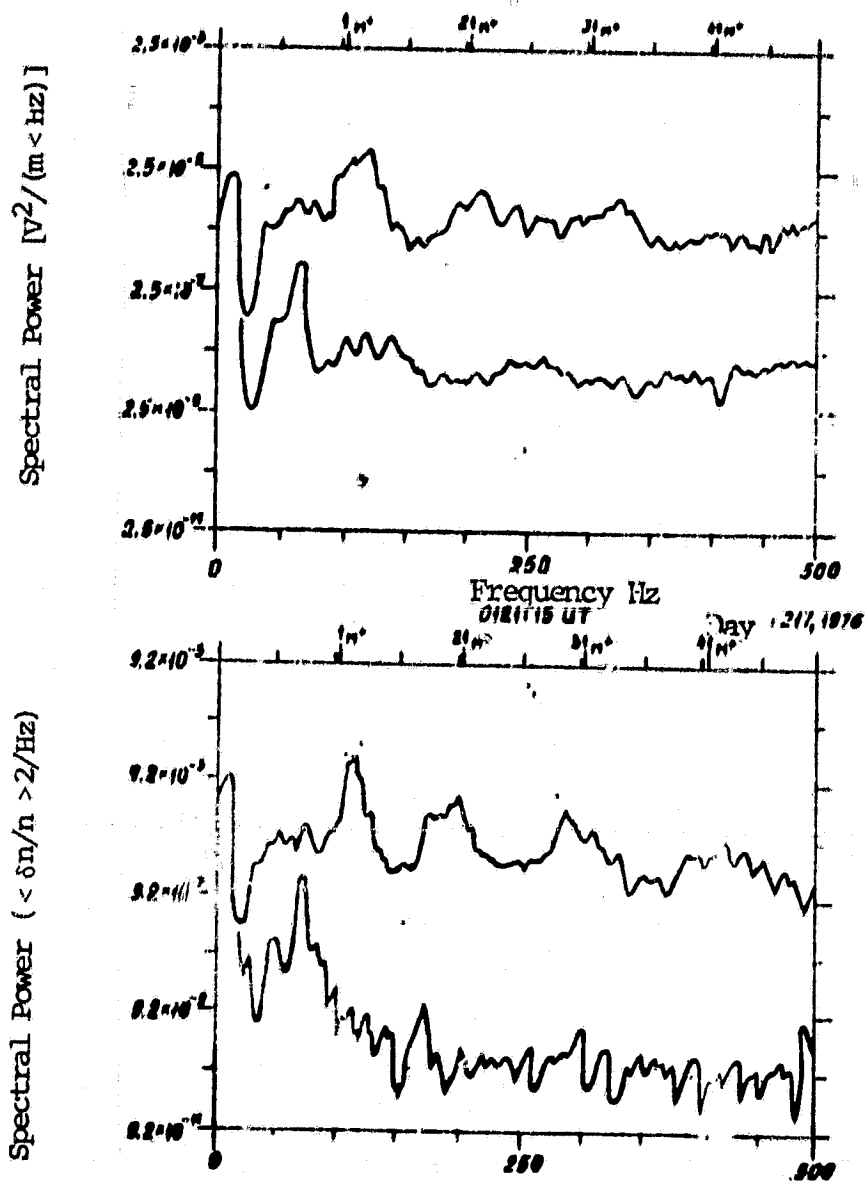Figure 1.1.   Example of Data of Measurements on
              S 3 - 3 of Spectral Powers $(\delta n/n)^2$
              and $(\delta E)^2$ on 4 August 1976 at 01.21.15
              UT.

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The average or typical values of the measured parameters are the following:

$$E_p \sim 10^{-3} V/M, \quad \delta E_\perp \sim 10 \ V/M,$$
$$J \sim 10^{-14} A/M^2, \quad K \sim 10 \ M^{-1}, \quad f \sim 100 \ Hz$$

Analysis of the findings permitted the authors of [24] to assert that the /8 observed turbulence is ion-cyclotron which corresponds to the Drummond-Rosenbluth mode. This conclusion was drawn based on the following arguments. First of all, in the spectra presented in Figure 1.1, there are clear maximums of spectral power at frequencies slightly greater than ω_{Bi}, $2\omega_{Bi}$, $3\omega_{Bi}$ as should be according to the linear theory of EIC-instability.

In one of the typical examples $\omega/\omega_{Bi} = 1.4$, and this, according to the linear theory (see Figure 1.4) corresponds to the ratio Te/Ti somewhat greater than one. Thus, although direct measurements of Te and Ti were not made, certain information regarding the ratio of temperatures was extracted by the authors of [24] from the set of experimental data and the hypothesis regarding the correctness of the linear theory of EIC-instability for this case. However, estimates for temperatures can hardly be considered experimental fact, since currently it is far from clear how much the conclusions of linear theory can be used to analyze the turbulent quasi-stationary state.

Secondly, in certain cases (when the time for passage of the region of wave electrostatic turbulence was greater than the period of rotation of the satellite around its own axis), polarization of low-frequency oscillations was defined. It was found that intensity of the variable electrical field is the maximum in the direction perpendicular to the main magnetic field B_0 with accuracy $\pm 15°$. This conclusion is an important confirmation of the fact that the observed waves correspond to the EIC-waves.

The authors of [24] estimated the temperature of the electrons T_e as follows. Since the quantities of fluctuations in the electrical field δE and density of electrons δn for slow waves are associated by the ratio

$$e\delta E n \cdot \nabla p = \nabla \delta n T_e \cdot k \delta n T$$ /9

(where k = wave number), then by using measurements of δn and δE one can determine the product of the wave number k times T_e ($kT_e \sim 5 \times 10^{-2}$ eV/m). The authors of [24], taking into consideration the Doppler effect in measuring frequencies from a moving satellite, based on the results of a theory developed in [46], estimated the phase velocity of waves and their wave number ($k \sim 1.6 \times 10^{-2}$ m^{-1}). This permitted further estimate of the temperature ($T_e \sim 3.4$ eV). The conclusion that the observed wave turbulence corresponds to the Drummond-Rosenbluth mode is also confirmed by the fact that with direct substitution of the obtained estimates for wave parameters in the actual part of the corresponding dispersion equation (see equation 2.1 of this survey) it is satisifed with a fair degree of accuracy.

It should be noted that these studies did not make measurements of fluctuations in the magnetic field, i.e., it was not experimentally proved that the waves only have electrostatic nature.

A shortcoming of the discussed observations is the fact that measurements of the variable electrical field were made only to frequencies ≤ 500 Hz (and in [44] data are presented to frequencies ≤ 1000 Hz), while the plasma ion frequency $\omega_{pi} \approx 2 \times 10^3$ Hz could be unnoticed because of the ion-sonic turbulence which possibly exists simultaneously with the cyclotron.

Thus, the set of wave experiments made on the satellite S 3-3 yields rich experimental information necessary to analyze the nature of the electrostatic wave turbulence in the magnetosphere. The cycle of works by the Mozer group to

investigate phenomena of anomalous resistivity, double layers and electrostatic
shockwaves based on these experiments and a number of hypotheses draws the conclu-
sion that the EIC-instability plays a definite role in these phenomena. However,
the set of experimental data, from our viewpoint, is still not sufficient for
reliable confirmation of the definitive role of the EIC-mode in the problem of
anomalous resistivity in the magnetosphere. One can only assert that the EIC-
turbulence really exists.

1.2. MAIN CONCLUSIONS OF THE LINEAR THEORY OF EIC-INSTABILITY

If one considers that the temperature of ions and electrons T_e and T_i for the
plasma in the magnetic field has a finite value which differs from zero, then one
can find the following effect [1, 2]. It is found that with $\theta \approx \pi/2$ (θ-angle between
direction of external magnetic field B_0 and wave vector $\vec{k}$) a number of new branches
of natural plasma oscillations appear whose frequencies with $k \to 0$ and $k \to \infty$ approach
the frequency $n|\omega_{B_0}|$ $(\mathcal{A} = 1,\epsilon; \; h = 1,2,... ; |\omega_{g_n}| = \epsilon B_0/m_{i}b)$. These oscillations are
called cyclotron.

The behavior of the cyclotron oscillations significantly depends on the proximity
of the angle θ to $\pi/2$. Therefore cases of transverse

$$\left(\theta = \frac{\pi}{2}, \; \kappa_{\shortparallel} = \kappa \cos\theta = 0, \; \omega \gg v_e k_{\shortparallel}, \; v_e = \sqrt{T_e/m_e} \right)$$

and quasitransverse dissemination of cyclotron oscillations are isolated. The condi-
tion of quasitransverse nature is written as: $k_{\shortparallel} v_i \ll \omega \ll v_e k_{\shortparallel}$.

We now assume that low-pressure plasma is examined for which the following condi-
tions are fulfilled (applied, in particular, to the magnetosphere)

$$4\pi n (T_i + T_e)/b_0^2 \ll \cos^2\theta \ll 1.$$

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It now turns out (see for example [27]), that from the ion-cyclotron oscillations it is possible to isolate almost electrostatic (potential, longitudinal) ion-cyclotron oscillations. If the latter conditions are not fulfilled, then the oscillations are not potential. In order to obtain electrostatic ion-cyclotron oscillations in the case of their quasitransverse dissemination, we will examine the dispersion equation for the Maxwellian functions of distribution which describe the longitudinal plasma oscillations in the magnetic field [27]:

$$\mathcal{E}_0 = 1 + \mathcal{E}_i + \mathcal{E}_e = 0$$

$$\mathcal{E}_\alpha = \frac{\omega_{e\alpha}^2}{k^2 v_\alpha^2}\left[1 + i\sqrt{\pi}\, z_0^\alpha \sum_{-\infty}^{\infty} e^{-\mu_\alpha} I_n(\mu_\alpha) W(z_n^\alpha)\right]$$

$$z_n^\alpha = (\omega - n\omega_{e\alpha})/\sqrt{2}\, k v_\alpha \cos\theta \,, \quad \mu_\alpha = (k_\perp g_\alpha)^2 \,,$$

$$g_\alpha = v_\alpha/\omega_{e\alpha} \,, \quad v_\alpha = \sqrt{T_\alpha/m_\alpha} \,,$$

$$W(z_n^\alpha) = e^{-z_n^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^{z_n^\alpha} e^{t^2}dt\right) ,$$

(1.1)

$I_n(\mu_\alpha)$--modified Besselian function. The function W is called the integral of probability from the complex argument or Cramp's function. In the case where the argument of the W function is great, or small as compared to a unit, this function can be approximated by the following approximate formula which follow from its determination [36]:

a) $|x| \gg 1$

$$W(x) = \frac{i}{\sqrt{\pi}\, x}\left(1 + \frac{1}{2x^2} + \frac{3}{4x^4} + \cdots\right) + e^{-x^2} ,$$

b) $|x| \ll 1$

$$W(x) = 1 + \frac{2i\, x}{\sqrt{\pi}} + \cdots \,.$$

Only the real arguments of the W function are further examined everywhere. Now in order to isolate the longitudinal ion-cyclotron oscillations in the case of their quasitransverse dissemination, we add the following to the hypotheses and conclusions made: $k_\perp g_i \sim 1$, $k_\perp g_e \ll 1$, $|\omega_{ei}| \gg |\omega|$, $|\omega_{ei}| \gg k_\parallel v_e$, $k_\parallel \ll k_\perp$,

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$$|\omega - n\omega_{bi}| \gg k_n v_i .$$

Then, after using the asymptotic formulas for $W(x)$, the contributions of ε_e and ε_i in the dispersion equation for longitudinal oscillations $\varepsilon = 1 + \varepsilon_e + \varepsilon_i = 0$ will be determined by the formulas:

$$\varepsilon_e = \frac{\omega_{pe}^2}{k^2 v_e^2} \left(1 + i\sqrt{\pi} \, z_e \right) \tag{1.2}$$

$$\varepsilon_i = \frac{\omega_{pi}^2}{k^2 v_i^2} \left[1 - \sum \frac{\omega}{\omega - n\omega_{bi}} \Gamma_n (k_\perp^2 g_i^2) \right] +$$

$$+ i\sqrt{\pi} \frac{\omega_{pi}^2}{k^2 v_i^2} \, z_e \sum \Gamma_n (k_\perp^2 g_i^2) \exp(-z_n^2) ,$$

$$\Gamma_n(\mu) = \exp(-\mu) I_n(\mu), \quad \mu = k_\perp^2 g_i^2, \quad z_e = \omega / \sqrt{2} k_n v_e,$$

$$z_n = (\omega - n\omega_{bi}) / \sqrt{2} k_n v_i, \quad \omega_{pa} = \sqrt{4\pi n e^2} / \sqrt{m_a} .$$

Only one antiHermitian term responsible for the Cerenkov absorption of oscillations by the electrons is left in the expression for ε_e, since according to the condition $|\omega_{bi}| \gg k_n v_e$, the electron cyclotron dampening of the examined oscillations has an exponential order of smallness. The condition $|\omega - n\omega_{bi}| \gg k_n v_i$ means that $k_n v_i / \omega_{bi} \ll n$ i.e., dissemination of the wave occurs almost perpendicularly to the magnetic field. At the same time, weak ion cyclotron damping of the oscillations is guaranteed. We will now assume for simplicity that $\omega_{pa}^2 / k^2 \gg 1$, when one can ignore in the dispersion equation the unit as compared to ε_i and ε_e. Then the equation for the real part of the oscillation frequency $(\rho_e \varepsilon_e, \omega_r, \vec{k}) = 0, \cdot k_e (l))$ in our case adopts the following appearance:

$$\frac{\omega_{pe}^2}{k^2 v_e^2} + \frac{\omega_{pi}^2}{k^2 v_i^2} \left[1 - \sum \frac{\omega \Gamma_n (\mu)}{\omega - n\omega_{bi}} \right] = 0$$

From here we obtain:

$$1 + \frac{T_e}{T_i} = \sum_{n=0} \frac{\omega \Gamma_n (\mu)}{\omega - n\omega_{bi}} + \Gamma \cdot (\mu) \tag{1.3}$$

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Since $\ddot{T} \cdots$, then
$$1 + \frac{T_i}{T_e} = 2\omega^2 \sum_{n=1}^{\infty} \frac{\Gamma_n(\kappa)}{\omega^2 - n^2\omega_{Bi}^2} + \Gamma_0(\kappa) \equiv f(\omega)$$

It is convenient to solve the equation $1 + T_i/T_e = f(\omega)$ graphically (see Figure 1.2 [27]). The solutions to $\omega^{(n)}(\kappa) = \omega$, $n = 1, 2 \ldots$ corresponds to the points of intersection of straight line $y = 1 + T_i/T_e$ and the curve $y = f(\omega)$.
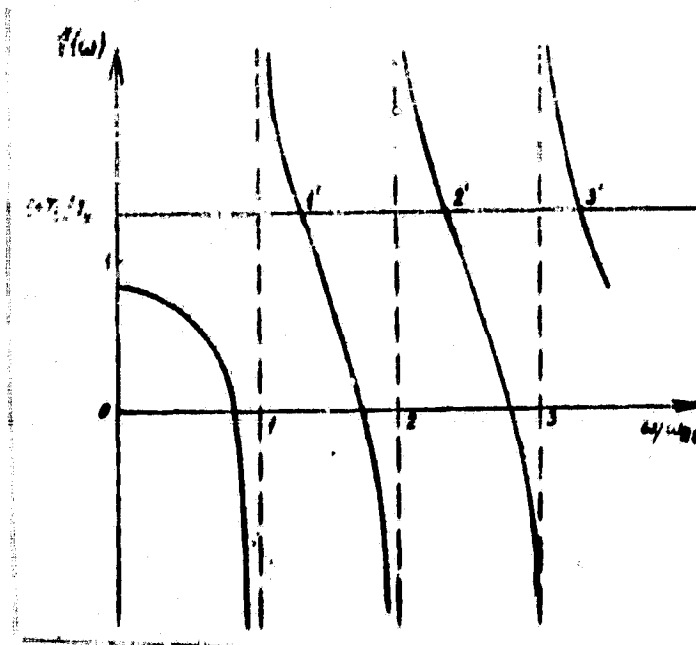


Figure 1.2. Graphic Resolution of the Dispersion Equation

It is apparent from Figure 1.2, that with a rise in T_i/T_e, $\omega^{(m)}(k)$ approaches $n\omega_{Bi}$. The behavior of the frequency $\omega^{(m)}(k)$ depending on "k" is schematically shown in Figure 1.3 [27]. It follows from Figure 1.3, that for long ($k\rho_i \ll 1$) and short ($k\rho_i \ll 1$) waves, the frequencies $\omega^{(m)}(k)$ close to $n\omega_{Bi}$. Therefore one can assume

$$\omega^{(n)}(\kappa) = n\omega_{Bi}(1 + \Delta_n(\kappa))$$

(1.4)

where $|\Delta_n(\kappa)| \ll 1$. In order to find $\Delta_n(k)$, we return to equation (1.3). By substituting instead of ω its expression (1.4), we bring (1.3) to the following appearance

$$1 + \frac{T_i}{T_e} = \sum_{m=-\infty}^{\infty} \frac{n}{n-m} \Gamma_m(\kappa) + \frac{\Gamma_n(\kappa)}{\Delta_n(\kappa)} + \Gamma_n(\kappa)$$
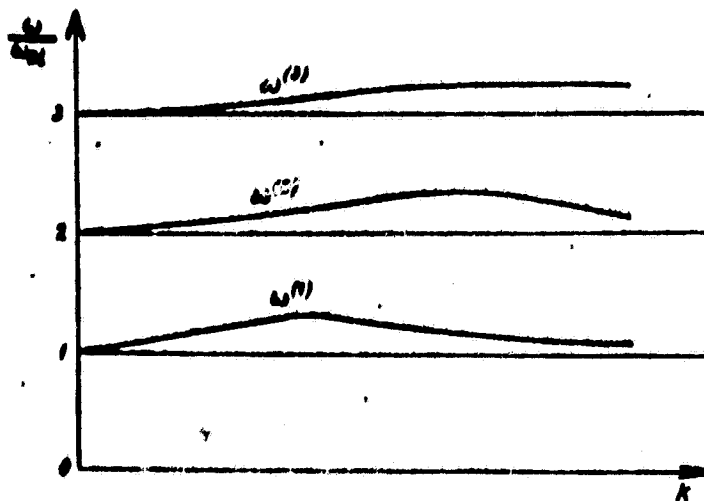
Figure 1.3.   Schematic Dependence of Frequencies
of Longitudinal Ion-Cyclotron Oscil-
lations on Quantity of Wave Vector

From here we find

$$\Delta_{\imath}(x) = \frac{\Gamma_{n}(\mu)}{1 + T_{\imath}/T_{e} - \left[\sum_{m \neq 0}\frac{n}{n+m}\Gamma_{m}(\mu) + \Gamma_{n}\right]}$$

(1.5)

We will limit ourselves in the further presentation only to the first harmonics

(n = 1). This limitation is justified by the fact that the weakly-dampening oscil-

lations with a large number of the branch "n" do not exist [27, 16].

Then for the first harmonics instead of (1.4) we have   (1.6)

$$\omega^{2} = \omega_{ei}^{2}\left(1 + \Delta_{1}(x)\right)$$

where

$$\Delta_{1}(x) = \frac{\Gamma_{1}(\mu)}{1 + T_{\imath}/T_{e} - \left[\sum_{m \neq 1}\frac{1}{1-m}\Gamma_{m}(\mu) + \Gamma_{1}\right]}$$

If we now total the infinite sum for "n," by using the properties of the functions

$\Gamma_{m}(\mu) : \Gamma_{n} = \Gamma_{-n} , \sum \Gamma_{n} = 1$ and the recurrent ratios for the Besselian function,

then

$$\Delta_{1}(x) = \frac{\Gamma_{1}(\mu)}{1 + T_{\imath}/T_{e} - G(\mu)}$$

(1.7)   /16

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where $G(\mu) \equiv \Gamma_1 + \dfrac{1 - \Gamma_0(\mu)}{\mu}$; $0 \leqslant G(\mu) < 1$. We will examine the area of applicability of formulas (1.6) and (1.7).

In the case of $T_i \gg T_e$, i.e., in plasma with hot ions and cold electrons (which, for example, occurs in the transitional region of the magnetosphere and in the area of diurnal polar cusps) we have
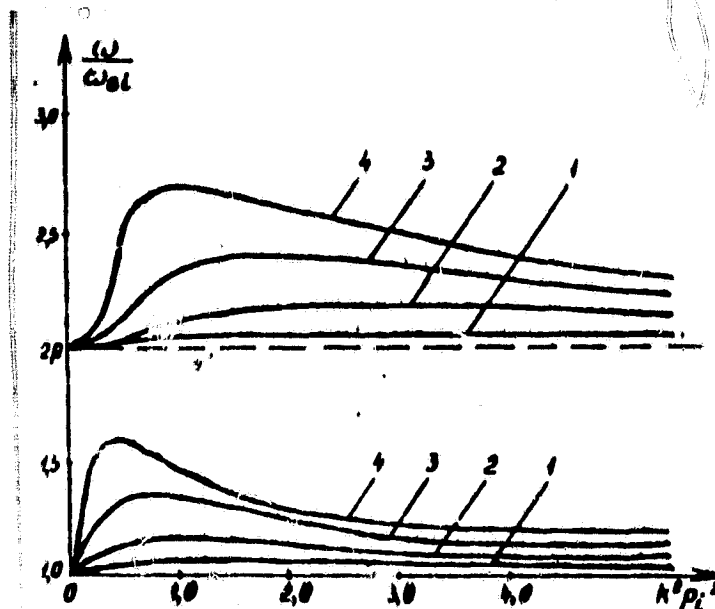
$$\omega^{(1)}(k) \approx \omega_{Bi}\left(1 + \frac{T_e}{T_i}\Gamma_1(\mu)\right). \tag{1.8}$$

We note that this formula expresses the proximity of $\omega^{(1)}(k)$ to ω_{Bi}, but now already with random values $\mu\left(\Gamma_1(\mu) \approx e^{-\mu}I_1(\mu)\right)$. Derivation of this usually employed formula is not equivalent to the result that is obtained if it is assumed $\mu = 0$ (then $G(o) = 1$, $1 - G = 0$, but $\Gamma_1(o) = 0$ and $\omega = \omega_{Bi}$). In the case of $T_i/T \approx 1$ for the maximum quantity $\Gamma_1 = 0.2$ which is reached with $\mu = 1.5$ [19], it is easy to see that $\Delta_1 \lesssim 0.2$, therefore $\omega = 1.2\omega_{Bi}$. When $T_e/T_i \gg 1$ and $\mu \to 0$ (so that $G(\mu) \to 1$), the correction Δ_1 can become large, since under these conditions $\omega^{(1)} \approx \omega_{Bi}\left(1 + \frac{\mu}{2}\frac{T_e}{T_i}\right)$. The numerical solution to equation (1.3) showed that the relationships for different ratios of temperatures have an appearance presented in Figure 1.4 [27]. The electrostatic ion-cyclotron fluctuations oscillate, for example, with drift of the electrons in relation to the ions. If the plasma electrons which have Maxwellian distribution for velocities drift under the influence of an external electric field along the external magnetic field B_0 with velocity u, then in the electron term in the dispersion equation (1.2) ω should be replaced by $\omega - k_n u$. The frequency of the longitudinal oscillation with $u \ll v_e$ is determined by formula (1.6) as before.

The expression for the increment in the oscillations in the linear theory is found from the formula

$$\gamma_k = -\frac{\text{Im}\,\varepsilon_0(\omega_k,\vec{k})}{\partial \text{Re}\,\varepsilon_0(\omega_k,\vec{k})/\partial\omega_k}$$

$$\omega_k = \text{Re}\,\omega(\vec{k}), \qquad \gamma_k = \text{Im}\,\omega(\vec{k})$$

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Figure 1.4. Dependency of Frequency and Longi-
tudinal Ion-Cyclotron Oscillations
on $(k^2 \rho_i^2)$ for Different Values of
the Parameter T_e/T_i. The values
T_e/T_i equal to 0.1, 0.3, 1 and 3
respectively correspond to curves
1 - 4.

Therefore, taking into consideration the small antiHermitian terms in (1.2) we

obtain for the first harmonic

$$\gamma = \gamma_e + \gamma_i = \sqrt{\tfrac{\pi}{2}} \, \tfrac{T_e}{T_i} \, \Gamma_1 \left\{ \omega_{B_i} \left(\tfrac{u}{v_e} - \tfrac{\omega}{k_n v_e} \right) - \omega \left(\tfrac{\omega - \omega_{B_i}}{k_n v_i} \right) exp \left[-\tfrac{1}{2} \left(\tfrac{\omega - \omega_{B_i}}{k_n v_i} \right)^2 \right] \right\}. \tag{1.9}$$

We now find from (1.9) an estimate for the critical drift velocity u_e which deter-

mines the threshold for development of the instability. In this case, $\gamma = 0$, there-

fore $\tfrac{u}{v_i} = \tfrac{\omega}{k_n v_i} \left[1 + \tfrac{v_e}{v_i} \tfrac{T_e}{T_i} \Gamma_1 exp \left[-\tfrac{1}{2} \left(\tfrac{\omega - \omega_{B_i}}{k_n v_i} \right)^2 \right] \right]$. By using (1.8), and by

designating $\xi = (\omega - \omega_{B_i})/\sqrt{2} k_n v_i$, we find that $\omega/k_n v_i = (1 + \Delta_1)\xi/\Delta_1$, therefore

$$\tfrac{u}{v_i} = \tfrac{1 + \Delta_1}{\Delta_1} \xi \left[1 + \tfrac{v_e}{v_i} \tfrac{T_e}{T_i} \Gamma_1 exp \left[-\tfrac{1}{2} \left(\tfrac{\omega - \omega_{B_i}}{k_n v_i} \right)^2 \right] \right] \equiv y$$

By minimizing the function $\Psi(\kappa_{\shortparallel}, \kappa_{\perp})$ in relation to $k_{\shortparallel}$, $k_{\perp}$, i.e., we solve the equations $\partial \Psi / \partial k_{\shortparallel} = 0$ and $\partial \Psi / \partial \kappa_{\perp} = 0$. As a result, we find that $\kappa_{\perp} \Psi_{\iota} \sim 1$ (so that $\Gamma_1 = 0.2$) and $(\omega - \omega_{\Phi_{\iota}})^2 / (\kappa_{\shortparallel} \varsigma_{\iota})^2 \approx 2 \ell_n \left(\varsigma_{\iota} T_{\iota} / \varsigma_{\iota} T_{\iota} \right)$

With these values $\kappa_{\perp} / \kappa_{\shortparallel} \approx 10$ and

$$U_c \approx \frac{\sigma_{\iota} T_{\iota}}{T_{\iota}} \, 10 \left(\ell_n (\sigma_{\iota} T_{\iota} \Gamma_{\iota} / \sigma_{\iota} T_{\iota}) \right)^{1/2} \tag{1.10}$$

The dependence of threshold (critical) drift velocity u_c on the temperature ratio T_e/T_i for hydrogen plasma is presented in Figure 1.5 [19]. It is apparent from the figure that in a fairly broad region of temperature ($0.1 < T_e/T_i < 8$), the EIC insta-bility is excited earlier (i.e., with smaller longitudinal points), than the ion-sound.
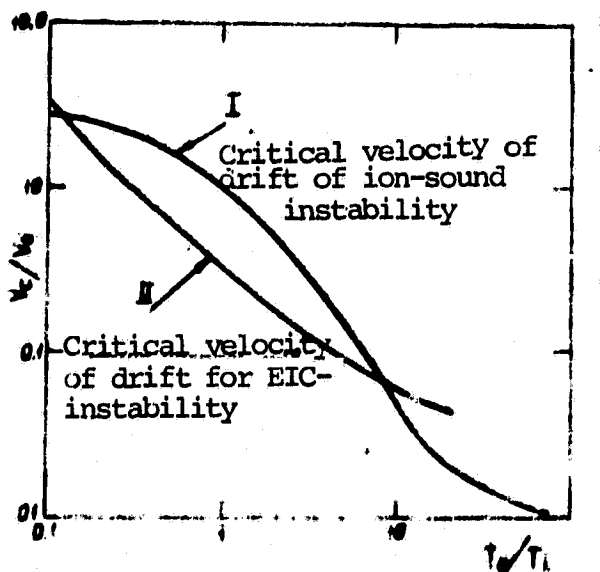
Figure 1.5. Dependence of Critical Velocity of Electron on Temperature Ratio T_e/T_i for Ion-Sound (I) and Ion-Cyclotron (II) Insta-bility

Figure 1.6 from [19] constructs the dependences ω/ω_{κ}, $\Gamma = (\kappa_{\shortparallel} \varsigma_{\iota})^2 / \kappa_{\perp}^2 \sigma_{\iota}^2 / \omega_{\rho_{\iota}}^2$ and $k_{\shortparallel}/k_{\perp}$ on T_e/T_i for hydrogen plasma (approximately the same will be for plasma containing ions of any other type). It follows from the Figure, for example, that

15
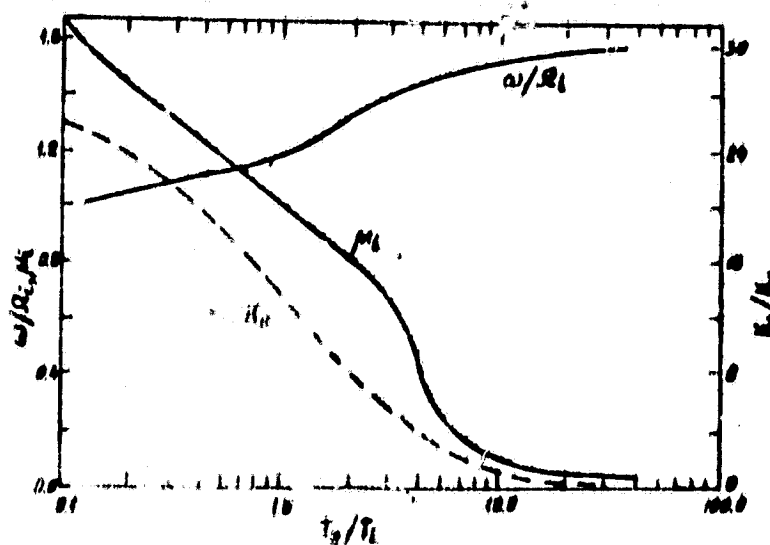
Figure 1.6. Critical Frequency Corresponding to
the Boundary of Instability ω/ω_{Bi},
Quantity $\mu_i = k^2 \; v_i^2/\omega^2_{Bi}$ and the
Ratio $k_\perp/k_\parallel$ Depending on T_e/T_i --
Temperature Ratios for Hydrogen
Plasma. ω/ω_{Bi} and μ_i correspond
to the same scale from the left,
$k_\perp/k_\parallel$ corresponds to the scale
from the right.

as T_e/T_i increases, the parallel component of the wave vector $k_\parallel$ of the increasing

waves rises (component along the field B_0), In this case $\omega \gg \omega_{Bi}$. Figures

1.5 and 1.6 refer to hydrogen plasma. Admixtures of heavier ions than H^+ change the

threshold for development of EIC-instability. Thus, for example, in $O^+ - H^+ -$

plasma instability with $\omega_{Bi}(O^+)$ develops at a lower threshold, even if the plasma

is hydrogen for 80% of the ion content [19].

We will summarize the presented linear theory of EIC-instability. It is evi-

dent that for studying the EIC-waves it is basically necessary to use a kinetic

equation, since the hydrodynamic theory of fluctuations in the magnetically-active

plasma is insufficient, since it does not suitably take into consideration the

thermal movement of the plasma particles. Because of consideration of thermal

movement of plasma particles, a number of new branches for oscillations in the

magnetically active plasma appear in the kinetic approximation [27, 26]: weakly- and strongly-damping, long- and short-wave, and all of them are missing in the cold plasma. In particular, an electrostatic ion-cyclotron wave mode of Drummond-Rosenbluth appears with frequency $\omega \approx n\omega_{Bi}$ and with wave vectors directed at angles close to $\pi/2$ in relation to the external magnetic field.

Theoretical study of the EIC-instability went beyond the framework of discussion of a laboratory experiment (to a considerable degree because of the work of Kindel and Kennel [19]), and its results began to be used more often to interpret phenomena in the auroral zone.

The development of EIC-instability at altitudes of the upper ionosphere, for calm conditions of the ionosphere, does not require very great density of the longitudinal currents (10^9 - 10^{10} electr./cm^2 x s, or 2 x 10^{-10} - 10^{-9} A/cm^2 [19]). Measurements in the auroral zone on satellites show the presence of longitudinal currents which are quite sufficient for oscillations of the EIC-waves (see for example [47]).

It is important to note that since the magnetic field is nonuniform because of longitudinal currents, the requirement of pressure balance results in the requirement of pressure gradient. The pressure gradient governs the drift instability with low threshold value of the current [30]. This effect is important with

$$\beta > 10^{-2} \ (\beta \approx \frac{4\pi n(T_i + T_e)}{B^2})$$ (i.e., for example, in the region of diurnal polar cusp), and consequently, is not suitable for the conditions of the upper ionosphere.

The EIC-waves themselves have been observed in the auroral zone [24], and simultaneously with ascending ions [44]. However, information on precisely which mechanism oscillates the EIC-waves (ion bundles or drifting electrons) has not been successfully obtained from the measurements [44]. If one considers the source of energy for the EIC waves to be ion bundles, then the question of obtaining these

bundles is still unresolved.

Because of the lack of an unequivocal solution to the question of the source of free energy for EIC-wave oscillation, it is useful to also examine the possible mechanism for generation of EIC-instability not in the plasma with current, but in the presence of counter flows of ions [31].

In the plasma layer of the magnetospheric tail, the development of the explosive phase of a substorm is accompanied by the appearance of hot plasma moving along the layer with average-mass velocity exceeding 600 - 1000 km/s [32, 33]. It is believed that these streams form in the region of formation of a neutral point at geocentric distances from the earth less than 10 R_E. Injection of plasma to the /23 earth among the geomagnetic field with velocities considerably exceeding the velocity of the transverse drift, can cause formation of a counterflow of plasma as a consequence of reflection of the first stream from the nearer points in the ionosphere. Under certain conditions, it is possible that different unstable modes will develop in this system, and in particular, electrostatic ion-cyclotron which is a probable cause of the development of anomalous resistivity in the auroral tube [15]. The results of satellite measurements [34] show that quasielectrostatic noise appears in the area of the plasma layer (as well as in the auroral tubes) simultaneously with recording of longitudinal proton streams.

Thus, the problem develops of determining the conditions for the development of EIC-waves in the plasma which is characterized by the presence along a uniform magnetic field of two counterstreams of hot "anisotropic" protons ($T_{\|i} \neq T_{\perp i}$) in the presence of "anisotropic" electrons ($T_{\| e} \neq T_{\perp e}$) (without consideration for the drift velocity of electrons along the magnetic field). Since the distribution of energetic electrons over the angles within the plasma layer remained isotropic at the moment the explosive phase of the substorm begins [48] one can therefore

consider $T_{e\parallel} = T_{e\perp}$.

Then the condition obtained in [31] for oscillation of the EIC-wave for the first harmonics looks like:

$$T_e / T_{\perp i} < 4 V^2 / \Gamma_1(\mu) \, v_e^2 \tag{1.11}$$

where V--drift velocity of the proton stream. The inequality (1.11) has the following physical meaning. Increase in the stream velocity of the protons along the magnetic field diminishes the threshold for development of EIC-instability. The parameters of the magnetospheric plasma during development of the explosive phase of the substorm satisfy the inequality (1.11) [31, 36]. Thus, the observed EIC-waves can oscillate in the magnetospheric plasma during interaction of the counterstreams of /24 hot protons with great perpendicular temperature.

Because of the importance of taking into consideration the temperature anisotropy in magnetospheric applications (for example, in the upper ionosphere where the plasma is collision-free), we will present formulas for this case [29]. Expression (1.8) for the frequency of EIC-fluctuations is transformed into

$$\omega^{(r)}(\kappa) = \omega_{bi} \left(1 + \frac{T_{e\parallel}}{T_{i\perp}} \Gamma_1(\mu) \right) \tag{1.12}$$

$$\mu = \kappa_\perp^2 \varrho_i^2 , \quad \varrho_i = v_{i\perp}/\omega_{bi}, \quad v_{i\perp} = (T_{i\perp}/m_i)^{1/2}$$

For the increment of EIC-instability, instead of (1.9) we have

$$\gamma = \sqrt{\tfrac{\pi}{2}} \frac{T_{e\parallel}}{T_{i\perp}} \Gamma_1(\mu) \left\{ \omega_{bi} \left(\frac{u}{v_{e\parallel}} - \frac{\omega}{\kappa_\parallel v_{e\parallel}} \right) - \right. \tag{1.13}$$

$$\left. - \omega \left(\frac{\omega - \omega_{bi}}{\kappa_\parallel v_{i\parallel}} \right) \exp\left[-\tfrac{1}{2} \left(\frac{\omega - \omega_{bi}}{\kappa_\parallel v_{i\parallel}} \right)^2 \right] \right\}$$
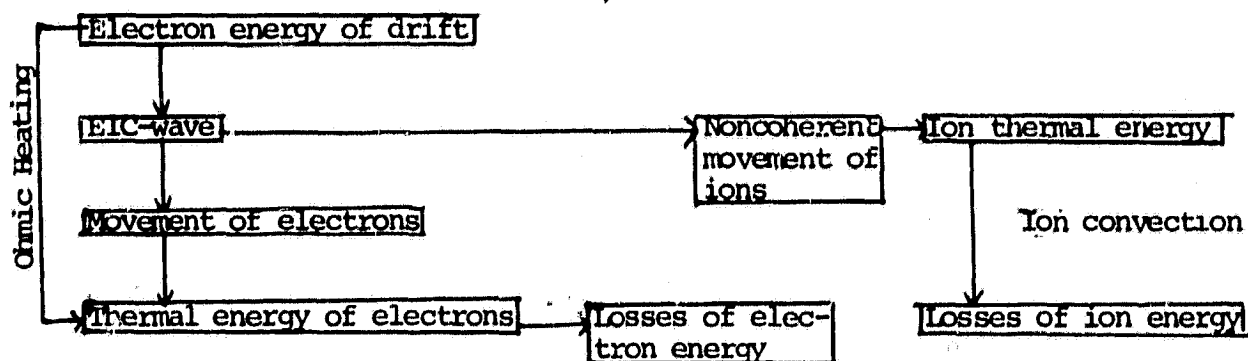
$$v_{i\parallel} = (T_{i\parallel}/m_i)^{1/2}, \quad v_{e\parallel} = (T_{e\parallel}/m_e)^{1/2}$$

Respectively, instead of (1.10) we obtain

$$u_c \simeq v_{i\parallel} \frac{T_{i\perp}}{T_{e\parallel}} 10 \left(\ell_n \frac{v_{e\parallel} T_{e\parallel}}{v_{i\parallel} T_{i\perp}} \Gamma_1 \right)^{1/2} \tag{1.14}$$

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1.3. EVOLUTION OF EIC-INSTABILITY

The general plan for evolution of EIC-instability excited by electron drift along the magnetic field can be presented as shown in Figure 1.7 [14].

Figure 1.7. Schematic Illustration of Evolution of EIC-Instability [14]

We note first of all that in its nature, the EIC-instability is weakly convective [21]. This circumstance is important for inhomogeneous conditions of the upper ionosphere where waves reach considerable amplitudes in the region of instability before wave convection from the instability region becomes important. In the region of maximum instability, the group velocity of waves transverse to the magnetic field approaches zero, while the group velocity along the magnetic field $v_{g\parallel}$ remains a finite quantity [21].

It is easy to show that by using (1.2) and the formula to determine

$$ v_{g\mu} \left(v_{g\mu} - \left(\frac{\partial \omega}{\partial k_\mu} \right)_\varepsilon = - \frac{(\partial \varepsilon/\partial k_\mu)_{\omega, k_\perp}}{(\partial \varepsilon/\partial \omega)_{\omega, k_\perp}} \right) $$

that $v_{g\mu} \lesssim v_i$. The slightly convective nature of the EIC-waves permits the instability to reach large amplitudes. In this case, the quasilinear effect of formation of a "plateau" on the distribution function of electrons which results in relaxation of the oscillations, in the opinion of the authors [20, 22] cannot play its role because of the spatial effects and

longitudinal electric fields occurring in the magnetosphere. Direct measurements on the satellite S 3 - 3 [15] also indicate that EIC-instability is saturated at a higher level than predicted by the quasilinear theory [1]. The amplitude of the EIC-waves therefore rises until nonlinear mechanisms of saturation begin to operate. One of these mecha\sms is broadening of the ion resonances [4]. In the opinion of the authors of [22], in addition to broadening the ion resonances, the mechanism of longitudinal capture of electrons is active, when a large number of electrons are trapped between the potential "walls" of the wave. This, in turn, results in anomalous resistivity to the current of electrons. The theory of broadening of ion resonances is based on the concept that as instability develops, plasma becomes turbulent. Orbits of ions moving in the turbulent plasma experience noncoherent perturbation because of the interaction of ions with stochastically changing components of the wave field. Consideration for this interaction results in distortion of the condition for particle resonance with the wave $(\omega - k_\parallel v_\parallel - n\omega_c \neq 0)$. This in turn, increases the number of particles which exchange energy with the wave.

Increase in wave damping on the ions results in replacement of the linear increment γ^L by the nonlinear γ^{NL}. This equals the linear increment supplemented by the term which corresponds to nonlinear damping $\Delta\omega$, i.e.,

$$\gamma^{NL} = \gamma^L - \Delta\omega \qquad (1.15)$$

The quantity $\Delta\omega$ is associated with wave amplitude $(\delta n/n)$ by the formula [4]

$$\frac{\Delta\omega}{\omega - \omega_{B_i}} = \left\{ \left[\frac{\delta n/n}{(\delta n/n)_{cr}} \right]^2 - 1 \right\}^{1/2} \qquad (1.16)$$

where

$$(\delta n/n)_{cr} = \frac{T_i}{T_e} \frac{1}{\mu} \left(\frac{\omega - \omega_{B_i}}{\omega_{B_i}} \right) \frac{1}{[F_n(\mu)]^{1/2}}$$

$$F_n(\mu) = \frac{1}{4} [J_n^2(\mu) + 2 J_n'^2(\mu) + J_n^2(\mu)], \quad \mu = k_\perp^2 \rho_i^2$$

In this case, the limitation of wave growth is determined by the condition

$$\gamma^{NL} = 0, \quad \Delta\omega_{SM} = \gamma^{L}$$ (1.17)

The quantitative characteristics for restriction in wave growth is the level of turbulence $\varepsilon_1 = W_E/nT_e$, where W_E—density of energy of electrical field of turbulent pulsations averaged for time. If in the expression for linear increment $\gamma^L, \gamma_e^L, \gamma_i^L$ (1.9), we ignore the term for resonance damping of a wave on ions, then the estimate of the turbulent level according to (1.16) and (1.17) is determined by the formula [4, 37, 12]:

$$\bar{\varepsilon}_1 = \frac{\overline{W_E}}{nT_e} = \frac{1}{n_e} \frac{1}{k_e} \sum_k \varepsilon_k^1 = 0.1 \left(\frac{\omega_{gi}}{\omega_{pi}}\right)^2$$ (1.18)

Experimental confirmation of the nonlinear stabilization of EIC-instability according to the Dum-Dupree mechanism [4] was obtained in laboratory experiments [16, 18].

The stage of nonlinear limitation in the growth of EIC-instability is followed by a stage of turbulent heating where the energy is transmitted from the wave to the particles. Wave energy density averaged for time (electrostatic plus kinetic) for the first harmonics equals [1]:

$$W = \frac{E_k^1}{8\pi} \left\{ 1 + \frac{1}{k^2\lambda_e^2} \left[1 + \frac{T_i}{T_e \Gamma_1} \right] \right\}$$ (1.19) /28

where $\lambda_e = v_e/\omega_{pe}$, λ_e —Debye radius of electrons. The first term in (1.19) corresponds to the electrostatic energy of the wave, the next two terms correspond to the kinetic energy of electrons and ions respectively. Just as, usually in the magnetosphere $k\lambda_e \ll 1$, so the kinetic energy of the particles is considerably greater than the energy of the field, while the ion kinetic energy is Γ_1^{-1} times greater than the kinetic energy of the electrons. Assuming that $E_k \sim \exp(\gamma t)$, the rate of energy transmission from directed movement of the electrons to kinetic energy of ions participating in wave movement, is written as

$$\frac{\partial W_i}{\partial t} = \frac{E_k^1}{4\pi} \frac{\gamma}{(k\lambda_e)^2} \frac{T_i}{T_e \Gamma_1}$$ (1.20)

We will now examine the process of energy transition from EIC-waves to thermal energy of ions in the framework of the quasilinear theory [9, 14]. We will define the heating rate as

$$\frac{\partial W_i}{\partial t} = \int d^3v \, \frac{1}{2} n m_i (v_\parallel^2 + v_\perp^2) \frac{\partial f_i}{\partial t} \qquad (1.21)$$

We initially focus attention on the "perpendicular" heating governed by resonance particles, i.e.,

$$\frac{\partial W_{i\perp}}{\partial t} = \int d^3v \, \frac{1}{2} n_i m_i v_\perp^2 \frac{\partial f_i}{\partial t} \qquad (1.22)$$

where the equation for the distribution function of ions f_i looks like:

$$\frac{\partial f_i}{\partial t} = \sum_k \sum_n \frac{\pi e^2}{m_i^2} \left(k_\parallel \frac{\partial}{\partial v_\parallel} + \frac{n\omega_{bi}}{v_\perp} \frac{\partial}{\partial v_\perp} \right) \left\{ \left(v_\parallel \frac{k_\parallel}{k} + \right. \right. \qquad (1.23)$$

$$+ \frac{n\omega_{bi}}{k} \right)^2 J_n^2(\mu_\perp) \, \delta(\omega_k - \omega_{bi} - k_\parallel v_\parallel) \frac{\langle E_k^2 \rangle}{\omega_k^2 m_i} \cdot$$

$$\cdot \left(k_\parallel \frac{\partial f_i}{\partial v_\parallel} + \frac{n\omega_{bi}}{v_\perp} \frac{\partial f_i}{\partial v_\perp} \right) \right\}, \quad \mu_\perp = k_\perp v_\perp / \omega_{bi}$$

Here ω_k-frequency of mode which corresponds to the linear theory; J_n--Besselian function. Further, the computations use the Maxwellian distribution function f_i and take into consideration only the first harmonics ($n = 1$).

By using (1.23), instead of (1.22) we obtain

$$\frac{\partial W_{i\perp}}{\partial t} = \frac{\pi \omega_{pi}^2 \omega_{bi}}{2 v_i^2} \sum \frac{\langle E_k^2 \rangle}{\omega_k^2} \int_{-\infty}^{\infty} dv_\parallel \, \delta(\xi)(k_\parallel v_\parallel + \qquad (1.24)$$

$$+ \omega_{bi}) f_\parallel \int_0^\infty dv_\perp \cdot v_\perp |Q^2| f_\perp ,$$

$$f = f_\perp \cdot f_\parallel , \quad |Q^2| = \left(\frac{k_\parallel v_\parallel}{k} + \frac{\omega_{bi}}{k} \right)^2 J_1^2(\mu_\perp) ,$$

$$\xi = \omega_k - \omega_{bi} - k_\parallel v_\parallel .$$

By integrating for v, using

$$\int_0^\infty v_\perp dv_\perp \, J_1^2(\mu_\perp) \exp\left(-\frac{1}{2} v_\perp^2 / v_i^2\right) = v_i^2 \, \Gamma_1(\rho) ,$$

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we obtain with $k_\perp \gg k_\parallel$

$$\frac{\partial W_{i\perp}^e}{\partial t} = \frac{\pi \omega_{pi}^2 \omega_{Hi}^3}{2(2\pi)^{1/2} v_i^3} \sum \frac{\langle \varepsilon_k^2 \rangle}{k^2 k_\parallel \cdot \omega_H} \overline{\Gamma_n(\mu)} \cdot$$

$$\cdot (1+\Delta_1)^2 \cdot \exp\left[-\frac{1}{2}\left(\frac{\Delta_1 \omega_{Hi}}{k_\parallel v_i}\right)^2\right] ,$$

(1.25)

where $\quad \Delta_1 = T_e \Gamma_n(\mu) / T_i$, $\quad \Delta_1 \omega_{Hi} = \omega - \omega_{Hi}$,

By using the expression for the term of damping on ions in (1.9), we obtain (for the average $\bar{k}$):

$$\frac{\partial W_{i\perp}^e}{\partial t} = \frac{T_i}{T_e \Gamma_n} |\gamma_k| \frac{\overline{\varepsilon_k^2}}{4\pi (\bar{k}\lambda_d)^2}$$

(1.26)

Similarly, for "parallel" heating

$$\frac{\partial W_{i\parallel}^e}{\partial t} = \frac{\pi \omega_{pi}^2 \omega_{pi}^3}{2(2\pi)^{1/2} v_i^3} \sum \frac{\langle \varepsilon_k^2 \rangle}{\omega_H k_\parallel k^2} \Delta_1 (1+\Delta_1)^2 \cdot$$

$$\cdot \Gamma_n(\mu) \exp\left[-\frac{1}{2}\left(\frac{\Delta_1 \omega_{Hi}}{k_\parallel v_i}\right)^2\right] \simeq \Delta_1 \frac{\partial W_{i\perp}^e}{\partial t} ,$$

(1.27)

where $\Delta_1 \sim 0.15$ for $T_i \sim T_e$, $\mu \approx 1.5$.

Thus, it is apparent from (1.26) and (1.27) that resonance heating of ions in a direction parallel to the magnetic field is an order smaller than the resonance heating of ions in a perpendicular direction.

We will now examine the nonresonance interaction of ions and waves for which instead of $\quad \delta(\omega_k - \omega_{Hi} - k_\parallel v_\parallel) \quad$ it is necessary to write the following in (1.23)

$$\frac{1}{\pi} \gamma_k / [(\omega_k - \omega_{Hi} - k_\parallel v_\parallel)^2 + \gamma_k^2]$$

(1.28)

where γ_k—increment in mode corresponding to the linear theory. With integration for $v_\perp$ in the region $(\omega_k - \omega_{Hi} - k_\parallel v_\parallel)^2 \approx \gamma_k^2$, we obtain for turbulent heating of nonresonance ions in a perpendicular direction (for average $\bar{k}$):

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$$\frac{\partial W_{ii}^{nt}}{\partial t} = \frac{T_i}{T_e T_i} \gamma_E \cdot \frac{E_i^2}{4\pi (E \lambda_0)^2}$$ (1.29)

Turbulent heating of nonresonance ions in a parallel direction (along the magnetic field) is exponentially small [14].

In this case one should recall that the effect of heating nonresonance particles is apparent, in the sense that (1.28) describes the response of the nonresonance particles to a change in wave amplitude. For example, when the wave amplitude increases, the oscillatory kinetic energy of the particles associated with the wave also rises, and the nonresonance particles seem to be heated [9].

Another situation develops when the resonance conditions $\omega_k - k_n v_u - n\omega_{0i} = 0$ are distorted because of the interaction of particles with turbulent pulsations [17]. Then the δ- function is replaced by

$$\frac{\gamma_E + k_\perp^2 D_\perp}{(\omega - k_n v_u - n\omega_{0i})^2 + (\gamma_E^2 + k_\perp^2 D_\perp)^2}$$ (1.30)

where γ_k—complete increment of wave increase $(\gamma_E = \gamma - \delta\omega)$, which equals zero in the state of saturated instability; $k_\perp^2 D_\perp$—transverse diffusion term. In the region $|\omega - k_n v_u - n\omega_{0i}| \leq k_\perp^2 D_\perp$ (i.e., in the resonance region) it is necessary to use the δ- function. Far from the resonance region, where $|\omega - k_n v_u - n\omega_{0i}| \gg k_\perp^2 D_\perp$ it is necessary to take (1.30), and $k_\perp^2 D_\perp$ is replaced by the average value [17]

$$\Delta\omega_i = \int k_\perp^2 D_\perp(\vec{v}) f_i(\vec{v}) d^3v$$ (1.31)

As a result, for EIC-instability in a state of saturation (with regard for (1.29)) one can obtain

$$\frac{1}{T_i}\frac{\partial T_i}{\partial t} = 9\left(\frac{3\lambda}{n}\right)_E \gamma_E$$ (1.32)

Formula (1.32) by magnitude yields the same rate of heating as the formula suggested in [37]:

$$\frac{1}{T_i}\frac{\partial T_i}{\partial t} = \frac{k_\perp^2}{2k^2}\gamma^2 = \frac{1}{2}\gamma^2$$ (1.33)

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However, (1.32) has a more common nature, since it is suitable for the case of wave amplitude saturation not only because of broadening of the ion resonances (formula (1.33) was obtained on this assumption), but also for possible saturation of the wave amplitudes with the help of convection effects in limited plasma.

Thus, one can state that distortion of the resonance conditions between velocities of the particles and phase velocity of the waves makes it possible for the greater part of the ion distribution function to exchange energy with the electrostatic ion-cyclotron fields. As a result, the rate of increase in ion temperature is proportional to complete linear increment γ^L. If we only examined the resonance particles, then the rate of heating ions would be proportional to $|\gamma_i^L|$.

We will now dwell on the question of the mechanisms for saturation of turbu- lent heating for EIC-turbulence. Since the first experiments which studied turbulent heating and its consequences were made under laboratory conditions with fairly small dimensions of the unit, therefore the corresponding theoretical constructions adopted the convective model for the heat emission mechanism, i.e., for saturation of turbulent heating. Correspondingly, the equation of thermal balance looked like [38]:

$$\left(\frac{\partial T_i}{\partial t}\right)_{iH} = \left(T_i - T_{i0}\right)\frac{v_{si}}{L}$$

L--dimension of the system. A similar approach in applying turbulent heating for saturation in longitudinal magnetospheric currents was developed in [37], where the following formula (1.33) was obtained for the velocity of turbulent heating of ions

$$\frac{1}{T_i}\frac{\partial T_i}{\partial t} = \left\{\gamma\right\}^L$$

. It was considered in this case that the actual EIC-instability is saturated by the mechanism for broadening of the ion resonances [4]. The quantity γ^L diminishes with a rise in ion temperature T_i (see (1.9)). Since instability occurs when $u > u_c$ see (1.14), while $u_{c\ell}$ correspondingly, rises with an increase

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in T_i, consequently, with a rise in ion temperature $u_c \to u$, while $f'(T_i)$, $\frac{\partial T_i}{\partial t} \to c$.
Thus, the effect of turbulent heating limits itself. In the condition on the boundary of stability, the "residual" turbulence heats the ions with such a rate that the processes of heat emission are capable of removing energy from the region of turbulence. In this case, one can write the equation for energy balance [37]:

$$\left(\frac{\partial T_i}{\partial t}\right)_{tg.} + \left(\frac{\partial T_i}{\partial t}\right)_{h.tr.} = 0, \quad \left(\frac{\partial T_i}{\partial t}\right)_{h.tr.} = \frac{T_i \, v_i}{L}$$

It should be stated that in the absence of heat emission, since turbulent heating in (1.33) is very rapid, EIC-instability will result in flashes of turbulence and   /33
local "spots" of hot ions [37, 38]. Thus EIC-instability under certain cases can play the role similar to the Bunemanovskiy instability, result in brief flashes of anomalous resistivity, and not lengthy quasistationary states.

The mechanism for heat emission which guarantees saturation of turbulent heating, may be different [13], where energy from $T_{i_\perp}$ jumps to $T_{i_\|}$ through the process of ion-ion collisions. In this case, if we consider that $T_{e_\|} = $ const, while the threshold of instability u_e approaches u with an increase in $T_{i_\perp}/T_{e_\|}$ because of "perpendicular" heating of ions (see (1.14)), then from an equation of balance for this mechanism we can obtain a level of turbulence for the second stage of evolution of EIC-instability, i.e., for turbulent "perpendicular heating [13]:

$$\varepsilon_1 = \frac{m}{e^2} \frac{\omega_{2i}}{n T_e} T_{i_\perp} \, v_{ii} \tag{1.34}$$

In this case $\varepsilon_1 < \varepsilon_1$. Experiments on S 3 - 3 [24, 15] forced us once again to attentively analyze the question of mechanisms for saturation of turbulent heating. In this case consideration of anisotropy of the temperatures of ions and electrons was basically important. The level of turbulence at the first stage of evolution, as shown above, is determined by formula (1.18):

$$\varepsilon_1 \approx 0.1 \left(\frac{\partial \omega_i}{\omega_{pi}}\right)^2$$

However, [15] drew the conclusion that the second stage of evolution ("perpendicular" turbulent heating) does not significantly change the quantity ε_1. This conclusion was drawn based on measurements on the satellite S 3 - 3, which showed that the measured level of turbulence ε is close to ε_1, while ε_1 was computed from measurements on the same S 3 - 3 according to formula (1.18). Therefore, decrease in the /34 level of EIC-turbulence ε_1 in the beginning of the second stage, which was expected according to the idea of "residual" turbulence [37] and according to formula (1.34) does not occur. Thus it was assumed in [15] that $\varepsilon \approx \varepsilon_1 \approx \varepsilon_2$. The physical interpretation of the conclusion that $\varepsilon_1 \cong \varepsilon_2$ is based on the fact that [15] the ratio $T_{i\perp}/T_{e\parallel}$ during turbulent heating remains almost constant, since both temperatures rise approximately at the same rate, so that u_c [see (1.14)] approaches u very slowly. Correspondingly, the level of turbulence ε_2 also slowly approaches a certain value ε_2^∞ which it adopts on the boundary of stability. Moreover, if in formula (1.34) the quantity ν_{ii} is replaced by $\nu_*^i \approx 0.1 \nu_*^e$, where $\nu_*^e \approx \omega_{Bi}$ [11], then one can obtain the quantity ε_2 on the order of ε_1 which coincides with the measured level of turbulence [15]. Thus, the authors of [15] draw the conclusion that decrease in the level ε_2 as compared to ε_1 is not a sufficient and significant circumstance for establishing the final level of turbulence.

In concluding the section on evolution of EIC-instability, one can make the following summarizations. EIC-instability evolution is customarily divided into several stages. The first stage is from the development of instability to its non-linear saturation at the corresponding level ε_1. Laboratory experiments which have studied EIC-instability excited both by currents [16, 18], and by ion bundles [23] have shown that saturation of wave amplitudes occurs according to the mechanism of broadening of the ion resonances [4]. Then follows the second stage of evolution associated with turbulent heating. At this stage "perpendicular" heating of ions

occurs. Corresponding laboratory measurements [17, 14, 38] prove the existence

of "perpendicular" heating of ions and agreement with theory. Because of heating   /3̲5̲

of ions, the threshold of instability u_c increases almost to u, and consequently,

the system approaches the boundary of stability (the condition of instability $u > u_c$,

where u is the drift velocity of electrons along the magnetic field). This self-

restriction of instability where the complete linear increment becomes very small

with a lower level of turbulence ε_2, correspondingly results in a slowing down of

the rate of turbulent heating. We note, however, that experiments on the satellite

S 3 - 3 indicate the possibility of such a situation where $\varepsilon_1 \approx \varepsilon_2$. The final stat-

ionary turbulent state is further defined from the energy balance in which the velo-

city of turbulent heating in a state close to the boundary of stability must be

balanced by the effective rate of heat emission. Appropriate laboratory experiments

[18] demonstrate considerable transfer of energy and particles perpendicularly to

the magnetic field, so that around the region with the current (i.e., around the

filament) a dense ring of hot ions is formed. In the actual filament, the density

in the number of particles is lower than in the surrounding ring of ions. The temper-

ature of ions within the filament is such that the gyroradius of ions corresponding

to it $\rho_i = v_i / \omega_i$ is on the order of or greater than cross dimensions of the fila-

ment. These same experiments indicate the appearance of suprathermal electrons dur-

ing instability in agreement with the measurements made on the satellite S 3 - 3

[15], and the presence of energetic electrons strongly depends on the magnetic

field in accordance with the theory of anomalous resistivity developed in [11].

The next section covers an examination of approaches to the problem of anoma-

lous resistivity governed by EIC-turbulence.

1.4. EIC-TURBULENCE AND
ANOMALOUS RESISTIVITY

Since the reason for EIC-instability is exceeding of a certain critical value

of current, then one can be convinced from general considerations that evolution

of instability must result in a mechanism which limits growth of the current. In

other words, an additional electrical resistivity must appear which is called anoma-

lous.

The standard formula for plasma resistivity $\eta = \frac{m_e \nu}{n e^2}$ contains the frequency

of collision (ν) of electrons with the scattering centers (ions, neutral atoms) in

relation to pulse loss. If the plasma electrons build up certain types of oscilla-

tions for waves as a consequence of instability, then anomalous loss of pulse occurs

(its transmission to oscillations, i.e., collective movements of ions). This loss

of pulse is characterized no longer by ν, but by a certain frequency ν_*. We will

determine ν_* according to the formula [14]:

$$n_e m_i \nu_* u = n_e m_e \int v_\shortparallel \frac{\partial f_e}{\partial t} d^3 v \qquad (1.35)$$

where for the velocity of change in electron distribution function f_e because of

interaction with waves one can write the following in the framework of the quasi-

linear theory [9]:

$$\frac{\partial f_e}{\partial t} = \sum_n \sum_k \frac{T_e^2}{m_e} \left(k_\shortparallel \frac{\partial}{\partial v_\shortparallel} + \frac{n \omega_{Be}}{v_\perp} \frac{\partial}{\partial v_\perp} \right) \left\{ \left(v_\shortparallel \frac{k_\shortparallel}{k} + \frac{n \omega_{Be}}{k} \right)^2 \right.$$

$$\times J_n^2 \frac{1}{\pi} \frac{\gamma_k}{(\omega_k - k_\shortparallel v_\shortparallel - n \omega_{Be})^2 + \gamma_k^2} \frac{\langle E_k^2 \rangle}{\omega_k^2 \cdot m_e}$$

$$\left. \cdot \left(k_\shortparallel \frac{\partial f_e}{\partial v_\shortparallel} + \frac{n \omega_{Be}}{k} \frac{\partial f_e}{\partial v_\perp} \right) \right.$$

For EIC-waves $\omega \sim \omega_{Bi} \ll \omega_{Be}$, and by leaving in (1.35) only the term with $n = 0$,

after integration for $v_\perp$ we obtain

$$n_e m_e \nu_* u = \sum_k \frac{\omega_{pe}^2}{4\pi v_e^2} \int_{-\infty}^{\infty} dv_{\shortparallel} \frac{k_{\shortparallel} \langle E_k^2 \rangle}{\omega_k^2} \left(\frac{k_{\shortparallel} v_{\shortparallel}}{k}\right)^2 \cdot$$

$$\cdot \frac{\gamma_k}{(\omega_k - k_{\shortparallel} v_{\shortparallel})^2 + \gamma_k^2} (\omega_k - k_{\shortparallel} u) f_{e\shortparallel} \Gamma_o (\mu_e) =$$

$$= \sum_k \frac{\omega_{pe}^2}{4\pi v_e^2} \frac{k_{\shortparallel} \langle E_k^2 \rangle}{\omega_k^2 k^2} (\omega_k - k_{\shortparallel} u) \Gamma_o (\mu_e) \cdot$$

$$\cdot \left[u + \frac{\omega_{Bi}}{k_{\shortparallel}} + \frac{i \gamma_k}{k_{\shortparallel}} + \sqrt{\pi} \frac{(\omega_k + i \gamma_k)^2}{k_{\shortparallel}^2} \cdot \right.$$

$$\left. \cdot Z \left(-\frac{u - \omega_{Bi}/k_{\shortparallel}}{\sqrt{2} v_e} + i \frac{\gamma_k}{\sqrt{2} k_{\shortparallel} v_e} \right) \right]$$

$$\mu_e = k_\perp^2 \, g_e^2 \, ; \quad g_e = v_e / \omega_{Be} \, .$$

where Z(x)—tabular function associated with the Kramp function determined above (W(x) by the correlation $Z(x) = i \sqrt{\pi} x \, W(x)$. We further consider that $\Gamma_o (\mu_e) \approx 1$ for $\mu_e \ll 1$; that $\left(\frac{u}{v_e} - \frac{\omega_{Bi}}{k_{\shortparallel} v_e} \right) \ll 1$, $\gamma_k \ll \omega_k$, and by examining only the first ion-cyclotron harmonics in ω_k, we obtained (using the asymptotics Z for small arguments):

$$m_e n_e \nu_* u = \sum_k \frac{\omega_{pe}^2}{4\pi k^2 \lambda_e^2} \frac{k_{\shortparallel} \langle E_k^2 \rangle}{\omega_k^2} (\omega_k - k_{\shortparallel} u) \sqrt{\frac{\pi}{2}} \omega_{Be}$$

$$- \frac{(u - \omega_{Bi}/k_{\shortparallel})^2}{2 v_e^2} \left\{ \frac{\omega_{Bi}}{k_{\shortparallel} v_e} - \Delta_1 \left(\frac{u - \omega_{Bi}/k_{\shortparallel}}{v_e} \right) + \right.$$

$$\left. + \Delta_1 \frac{\omega_{Bi}}{k_{\shortparallel} v_e} \left(\frac{u - \omega_{Bk}/k_{\shortparallel}}{v_e} \right) + 2 \Delta_1 \frac{\omega_{Bi}}{k_{\shortparallel} v_e} \left(\frac{u - \omega_{Bi}/k_{\shortparallel}}{v_e} \right)^2 \right\} .$$

The first term in the brackets provides a contribution from the resonance particles. It is an order greater than the contribution from the nonresonance particles which

describes the remaining terms in the brackets, since $\Delta_1 \sim 0.15$ for $T_e \sim T_i$ [14].

By leaving only the first term in the brackets, we obtain

$$m_e n_e v_* u = \sum_n \frac{\langle E_k^2 \rangle}{4\pi(k\lambda_e)^2} \left(\frac{k_\parallel}{\Delta_1 \omega_{Bi}} \right) \gamma_k^2.$$

Finally, for the average E we have:

$$v_* \simeq \frac{\varepsilon}{\varepsilon_d} \left(\frac{u}{\Delta_1 \omega_{Bi}/k_\parallel} \right) \frac{\gamma_k^2}{2}; \quad \varepsilon_d = \frac{1}{2} n_e m_e u^2,$$

where ε --level of turbulence, while γ_k^2 corresponds to the first term in (1.9).

Consequently, the effective quantity of anomalous resistivity [14]:

$$\eta = \frac{m_e v_*}{ne^2} \simeq 2\pi \frac{\varepsilon}{\varepsilon_d} \frac{u}{(\Delta_1 \omega_{Bi}/k_\parallel)} \frac{\gamma_k^2}{\omega_{pe}^2}$$

We note that turbulent "perpendicular" heating of ions mainly occurs because of nonresonance particles, while the effective frequency of collisions v_* which determines the pulse loss during collective collisions is governed by resonance particles. The quantity of anomalous resistivity for EIC-instability which is in a state of saturation (according to the mechanism for broadening of ion resonances [4]) with level ε_1 0.1 $(\omega_{Bi}/\omega_{pi})^2$ was computed in [12]. The authors started from the first moment in the equation

$$-\frac{e}{m_e} E_\parallel \frac{\partial f_e}{\partial v_\parallel} - \frac{e}{m_e} \left(\frac{v_\perp B_0}{c} \right) \frac{\partial f_e}{\partial v_\perp} = \frac{\partial}{\partial v_\parallel} D_\parallel^e (v, \varepsilon_1) \frac{\partial f_e}{\partial v_\parallel}$$

which yields

$$E_\parallel = -\frac{m_e}{ne} \int d^3 v \, D_\parallel^e (v, \varepsilon_1) \frac{\partial f_e}{\partial v_\parallel}. \tag{1.36}$$

Here $D_\parallel^e$--diffusion coefficient in the space of velocities, ε_1--mentioned level of turbulence. Since (see for example [12]):

$$D_\parallel^e = D_{\parallel\parallel}^e = \frac{e^2}{m_e^2} \sum_n \int d^3 k \frac{\langle E_k^2 \rangle J_n^2 \left(\frac{k_\perp v_\perp}{\omega_{Be}} \right)}{i(-\omega + k_\parallel v_\parallel + n\omega_{Be})} \cdot \frac{k_\parallel^2}{k^2},$$

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where J_n--Besselian function on the order of n, while

$$\frac{1}{\omega - k_\parallel v_\parallel - n\omega_{Be}} = \mathcal{P}\frac{1}{\omega - k_\parallel v_\parallel - n\omega_{Be}} + i R_e(D_\parallel^e)$$

then, in order not to solve the integral equation for $D_\parallel^e$, the resonance function of electrons was approximated by the Gaussian

$$R_e \simeq \frac{1}{(2\pi k_\parallel^2 \beta_{1/2}^2)^{1/2}} exp\left[-(k_\parallel v_\parallel - \omega)^2 / 2 k_\parallel^2 \beta_{1/2}^2\right]$$

$$\beta_{1/2} \sim \frac{1}{k_\parallel}\left(k_\parallel^2 D_\parallel^e(v_c, \varepsilon_1)/3\right)^{1/3},$$

where $\beta_{1/2}$--half-width of resonance, v_c--critical velocity at which the diffusion coefficient has the maximum $(\bar{\omega} - k_\parallel v_c) \sim (k_\parallel^2 D_\parallel^e(v_1, \varepsilon_1))^{1/3}$

After placing an expression for $D_\parallel^e$ in (1.36) and integrating for velocities on the assumption of Maxwellian distribution for f_e, in [12] the following was obtained for the quantity of anomalous resistivity

$$\eta = 0.06\left(\frac{\omega_{Bi}}{\omega_{pi}}\right)\left(1 - 12\frac{v_c}{u}\right)\frac{1}{\omega_{pe}}$$

and consequently,

$$v_* = \frac{1}{4\pi} 0.06 \frac{\omega_{Bi}}{\omega_{pi}}\left(1 - 12\frac{v_c}{u}\right)\omega_{pi} \simeq 0.1\omega_{Bi}$$

The same quantity v_* in a quasilinear approximation was obtained in an earlier work [49].

The ion-cyclotron mode of Drummond-Rosenbluth as a claimant for the main role in the problem of anomalous resistivity of the magnetosphere obtained powerful support after publication of the measurement results on the satellite S 3 - 3 [15, 24]. These measurements proved the presence of the attained high level of EIC-turbulence

$$\left(\varepsilon = T_\perp^i / 8\pi n_i T_e \simeq 3 \cdot 10^{-4}\right).$$

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This permitted the authors of [15] to assert that in the observed regions of EIC-turbulence, the "plateau" on the distribution function of electrons is not formed, since for the measured level, the ratio $e\Psi/T_e \ll 1$, while the quasilinear theory predicts $e\Psi/T_e \sim 10^{-2}$. Anomalous resistivity based on measured parameters of turbulence was computed in [15], based on the law for pulse preservation in the electron-wave system

$$m_e u \dot{v}_e = e E_{\parallel}$$

$$\nu_e = -\frac{e\langle \xi \dot{u} \rangle}{m_e u^2} = -\frac{e^4}{m_e^2 u^2 \omega_{pe}^2} \int \frac{dk}{(2\pi)^3} d\omega (\vec{k}\vec{u}) \langle E_{k\omega}^2 \rangle I_m \varepsilon_e$$

where $I_m \varepsilon_e$—imaginary part of electron dielectric function [28]. Adopting an approximation for random phase during integration and monochromatic spectrum with $k = E$, one can obtain $\nu_e = -2 \bar{E}_k I_m \varepsilon_e E_k^2 / 8\pi m_e n u$ where integration for $d^3 Rd$ was done from $\langle E_{k\omega}^2 \rangle$ to E^2_k. By using the value $I_m \varepsilon_e$ (see (1.2)), we have:

$$\nu_e = \frac{2\bar{E}_{\parallel}}{8\pi k m_e u} E_k^2 \frac{\sqrt{\pi}}{(k\lambda_e)^2} \left(\frac{u}{v_e} - \frac{\omega}{k_{\parallel} v_e} \right)$$

After taking, according to the measurements,

$$k_{\parallel}/k_{\perp} \sim 0.1, \quad k_{\perp} \sim 5 \cdot 10^{-4} \, cm^{-1}, \quad T_e \sim 1 \, \mathrm{эВ},$$

$$v_e \approx 6 \cdot 10^{4} \, cm/c, \quad \omega_{Bi} \sim \omega \sim 2\pi \cdot 10^{2} \, c^{-1},$$

$j_{\parallel} = 10^{-5}$ A/m^2 u = 6 × 10^7 cm/s, we find that $\nu_e \sim \omega_{Bi}$, while corresponding /41 anomalous resistivity $\eta \approx 1 \times 10^2$ Ohm × m (while the resistivity governed by particle collision $\sim 10^{-3}$ Ohm × m).

Field intensity governed by anomalous resistivity $E_{\parallel} \sim 2 \times 10^{-3}$ V/m. It should be said that these estimates in [15] were made on the assumption that the ion-sound turbulence is missing, although with u $\sim v_e$, the ion-sound or Bunemanovskiy turbulence must be perturbed. This circumstance was indicated in publications [39, 40]. In addition, accuracy of the estimate of the cofactor $k_{\perp}/k_{\parallel}$ which is included in the formulas is not great and the result can easily be exaggerated or underestimated by an order.

The obtained formulas for anomalous resistivity were based on mechanisms of saturation which did not take into consideration the boundaries of the field tubes in the magnetosphere. Since, according to current opinion (see for example [40]), microwave structure of the longitudinal currents is possible, we will present an estimate of anomalous resistivity from [37] which takes into consideration the boundaries.

From the equation $\frac{1}{2}\left(n_e \frac{\partial T_e}{\partial t}\right)_{u_e} = j_n E_n = \eta j_n^2$ where $\eta = \sqrt{4\pi m_e}/n e^2$, while $\left(\partial T_e / \partial t\right)_{u_e} = \left(\partial T_i / \partial t\right)_{u_i} = T_i v_i / L$ L—dimension of field tube, the following was obtained $v_e \leq \left(\frac{u_a}{u}\right)\left(\frac{T_i}{T_e}\right)\left(\frac{v_i}{L}\right)$. Anomalous resistivity in this case according to the estimates in [37] is only an order greater than the quantity of resistivity because of particle collisions.

2. MODEL OF INCLINED-CYCLOTRON DOUBLE POTENTIAL LAYER

### 2.1. ONE-DIMENSIONAL MODEL OF INCLINED LAYER

We will examine the following problem. Assume that the electric current flows upwards along a uniform magnetic field directed along the axis Z (Figure 2.1). We will consider that in the space V_O (i.e., above and to the right of the double layer) the directed velocity of thermal ions is negligible

$$\left(m_i u_i^2\right)_{V_0} \ll \left(T_i\right)_{V_0} \tag{2.1}$$

while, the electrons move downwards with fairly great directed velocity. The electrostatic potential on the upper boundary of the layer equals ϕ_0. In the lower half-space (V_1), on the contrary, it is assumed that the electrons are fixed:

$$\left(m_e u_e^2\right)_{V_1} \ll \left(T_e\right)_{V_1} , \tag{2.2}$$

while the ions are moving upwards with relatively great velocity. The potential on the lower boundary is assumed to be equal to ϕ_1.
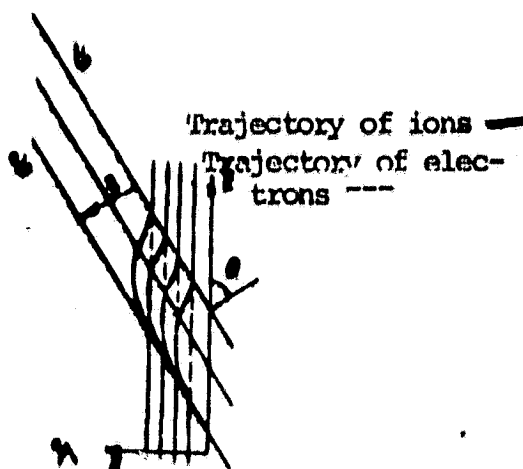
Figure 2.1. Calculation of Inclined Double Layer

Correspondingly, the index (zero) will mark the particles entering the layer from above, while the index (1) will indicate the particles entering it from below. It is assumed in this case that the parameters of the plasma and the field along the layer (i.e., along the y-axis in Figure 2.1) do not change ($\partial / \partial y = 0$).

Distribution of the electrostatic potential ϕ in the layer is determined by Poisson's equation:

$$\nabla^2 \phi(\vec{r}) = -4\pi e \left[n_i(\vec{r}) - n_e(\vec{r}) \right] \tag{2.3}$$

If the layer is strong, then in the layer $n_{i_0} \ll n_{i_1}$, $n_{e_0} \gg n_{e_1}$,

$$\nabla^2 \phi(\vec{r}) = -4\pi e \left[n_{i_1}(\vec{r}) - n_{e_0}(\vec{r}) \right] \tag{2.3a}$$

where $n_{i,e}$ --concentration of ion and electron, respectively equal to

$$n_{i,e}(\vec{r}) = \int f_{i,e}(\vec{u}, \vec{r}) \, d\vec{u} \tag{2.4}$$

and $f_{i,e}(\vec{u}, \vec{r})$ --distribution functions of particles and velocities. Assuming that the distribution function does not depend on the phase of the cyclotron rotation of

the particle, we rewrite (2.4) in the form

$$n = 2\pi \int f(u_\parallel, u_\perp) u_\perp \, du_\perp \, du_\parallel \approx \int F(u_\parallel) \, du_\parallel \, , \tag{2.4a}$$

where $f(u_\parallel, u_\perp)$--distribution function of particle of corresponding type (the sign i or e is omitted for brevity of writing), satisfying the Vlasov equation, in a stationary case having the following appearance:

$$\vec{u} \frac{\partial f}{\partial \vec{r}} + \frac{e}{m}\left(-\frac{\partial}{\partial \vec{r}} \phi + \frac{1}{c}[\vec{u}\vec{B}]\right)\frac{\partial f}{\partial \vec{u}} = 0 \tag{2.5}$$

As is known, solution to equation (2.5) can be written in the form of a random function from integrals of particle movement [56]. This function is completely determined by assigning the boundary conditions. In order to search for the necessary movement integral in this case, (at least two) we will use the circumstance that in the case where the transverse dimensions of the layer are much greater than the Larmor radius (we will be convinced of the correctness of this hypothesis later), particle movement can be described in the framework of drift approximation. Then, as Swift showed [52], one can introduce the following movement integrals:

$$k_1 \equiv e\phi + \frac{1}{2} m u_\parallel^2 + \frac{1}{2} m c^2 \left(\frac{\nabla_\perp \phi}{B}\right)^2 \approx e\phi^H + \frac{1}{2} m \left(u_\parallel^H\right)^2 \, , \tag{2.6a}$$

and with consideration for the inertia drift of particles,

$$k_2 \equiv u_\perp \left(1 + \frac{e}{m\omega_B^2}\nabla_\perp^2 \phi\right)^{-1/2} \approx u_\perp^H \, , \tag{2.6b} \quad \underline{/45}$$

where u^H--initial velocity of particle movement in the layer ($u^H = u_0$ for electrons, $u^H = u_1$ for ions); respectively $\phi^H = \phi_0$ for electrons and $\phi^H = \phi_1$ for ions; ω_B-- gyrofrequency of particles; $\frac{i\times e}{B^2} = \vec{v}_d$ and $\frac{\times e}{c} = \vec{v}_\Delta$ at the point of location of the leading center of the particle. By using the movement integrals (2.6a) and (2.6b), we will write a solution to the equation (2.5) in the form

$$f(u_\perp, u_\parallel) = f^H\left(u_\perp^H, u_\parallel^H\right) \tag{2.7}$$

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In fact, by having the movement integrals as an argument, this function satisfies equation (2.5), and coinciding with the initial distribution function of frequency for velocity assigned on the layer boundaries, it also satisfies the boundary conditions.

By substituting (2.7) in (2.4a), we obtain

$$n = 2\pi \int f^H(u_\parallel^H, u_\perp^H) \, u_\perp \, du_\perp \, du_\parallel \; , \tag{2.8}$$

where the quantities $u_\perp$ and $u_\parallel$ are linked by the quantities $u_\perp^H$ and $u_\parallel^H$ by the correlations (2.6), from which, it follows in particular that

$$u_\perp du_\perp = \left(1 + \frac{e}{2m\omega_B^2} \nabla_\perp^2 \phi\right) u_\perp^H du_\perp^H \tag{2.9a}$$

$$du_\parallel = \frac{u_\parallel^H du_\parallel^H}{u_\parallel} = \frac{u_\parallel^H du_\parallel^H}{\left[(u_\parallel^H)^2 + \frac{2e}{m}(\phi^H - \phi) - \frac{c^2(\nabla_\perp \phi)^2}{B^2}\right]^{1/2}} \tag{2.9b}$$

If the velocity of electric drift is small as compared to the velocity of longitudinal movement $u_\parallel$, expression (2.9b) is simplified and adopts the following appearance: /46

$$du_\parallel = \frac{u_\parallel^H du_\parallel^H}{u_\parallel^*} \left(1 + \frac{1}{2} \frac{c^2(\nabla_\perp \phi)^2}{(u_\parallel^*)^2 B^2}\right) , \tag{2.10a}$$

$$u_\parallel^* = \left[(u_\parallel^H)^2 + \frac{2e(\phi^H - \phi)}{m}\right]^{1/2} , \tag{2.10b}$$

$u_\parallel^*$ is the velocity which particles would have if the electrical field was strictly parallel to the magnetic field. By substituting expressions (2.9a), and (2.10a) into (2.8), we find the particle concentration:

$$n = \left(1 + \frac{e}{2m\omega_B^2} \nabla_\perp^2 \phi\right)^2 \left(n^* + \frac{1}{2} \frac{c^2(\nabla_\perp \phi)^2}{B^2} \int \frac{F(u_\parallel^H) u_\parallel^H du_\parallel^H}{(u_\parallel^*)^3}\right) \tag{2.11}$$

$$F(u_\parallel^H) = 2\pi \int f^H(u_\parallel^H, u_\perp^H) u_\perp^H du_\perp^H .$$

$$n^* = \int \frac{u_\parallel^H F(u_\parallel^H)}{u_\parallel^*} du_\parallel^H$$
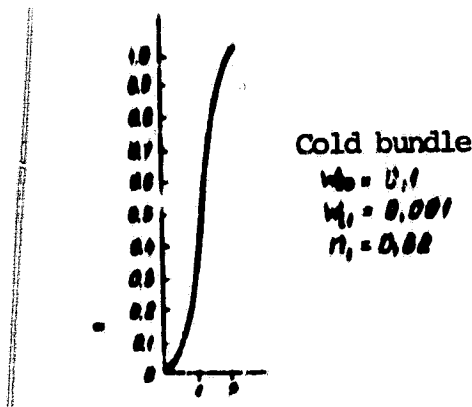
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Figure 2.2. Distribution of Electric
Potential Transverse to
Inclined Double Layer in
Cold Plasma

which is the density of particles corresponding to the electrical field $E_\perp = 0$.

It follows from expression (2.12b) and (2.10b) that

$$\nabla_\perp n^* = -\int \frac{u_{\shortparallel}^\mu \nabla_\perp u_\perp^* F(u_{\shortparallel}^\mu) \, du_{\shortparallel}^\mu}{(u_{\shortparallel}^*)^2} = \tag{2.13}$$

$$= \frac{e}{m} \nabla_\perp \phi \int [u_{\shortparallel}^\mu F(u_{\shortparallel}^\mu) / (u_{\shortparallel}^*)^3] \, du_{\shortparallel}^\mu$$

and the final expression for ion density looks like

$$n_i = \left(1 + \frac{c}{2m\omega_{B_i}^2} \nabla_\perp^2 \phi\right)^2 \left(n_i^* + \frac{1}{2} \frac{m_i c^2}{e B^2} \nabla_\perp n^* \nabla_\perp \phi\right) \tag{2.14}$$

As for the electrons, because of their relatively small mass, the corrections associated with their inertia drift in a nonuniform electric field (in expression (2.14), the terms proportional to $\nabla_\perp^2 \phi$ and $\nabla_\perp \phi$) are negligible and $n_e = n_e^*$ (2.15)

By substituting the expressions obtained for particle concentration in the Poisson's equation (2.3), we obtain

$$\nabla^2 \phi - \nabla_\perp^2 \phi + \left(1 + \frac{4\pi n_i^* e^2}{m_i \omega_{B_i}^2}\right) \nabla_\perp^2 \phi + \frac{1}{2} \frac{4\pi m_i c^2}{B^2} \nabla n_i^* \nabla \phi = -4\pi e(n_i^* - n_e) \tag{2.16}$$

Taking into consideration that in the examined case $v_\perp^2 = \frac{\partial^2}{\partial x^2}$ and $v^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$,
and by introducing the Alven dielectric permeability $\epsilon_a = 1 + c^2/v_a^2$ where $v_a^2 = B^2/4\pi n_i^* m_i$,
expression (2.16) can be rewritten in the form:

$$\frac{\partial^2 \varphi}{\partial z^2} + \epsilon_a \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{2} \frac{\partial \epsilon_a}{\partial x} \frac{\partial \varphi}{\partial x} = -4\pi e (n_i^* - n_e) \tag{2.17}$$

However, the coordinates x and Z, introduced as is shown in Figure 2.1, are
not very convenient for analysis of the examined task, since the boundaries of the
examined layer in this system are not coordinate lines. In this respect, we will
introduce a new variable $\eta = z - \alpha x$, where $\alpha = tg\theta$ (see Figure 2.1). In these
coordinates the boundaries of the layer are represented by the planes $\eta = 0$ and $\eta =$
η_1. In this case, the final expression for Poisson's equation look like:

$$(1 + \alpha^2 \epsilon_a) \frac{\partial^2 \varphi}{\partial \eta^2} + \frac{\alpha}{2} \frac{\partial \epsilon_a}{\partial \eta} \frac{\partial \varphi}{\partial \eta} = -4\pi e (n_i^* - n_e) \tag{2.18}$$

or

$$\frac{d}{d\eta} \left[\left(m + \alpha^2 \epsilon_a \right) + \ldots \right] = 8\pi e (n_i^* - n_e), \tag{2.18a}$$

where

$$n_i^*, n_e = \ldots$$

Equation (2.18a) coincides with the known equation which describes the poten-
tial distribution in the double layer in the absence of an external magnetic field
(see for example, in equation (3.6) from [52]) with only the difference that in the
left side of (2.18a) there is the multiplier $(1 + \alpha^2 \epsilon_a)$ which is usually much /48
greater than a unit. The quantity n_i^* and n_e is computed, as we have seen, as if
the component of the electrical field which is transverse in relation to the kinetic
field were missing, that is, in the same way as in the case of a direct double layer.
This circumstance makes it possible, without repeating the mathematical computations,
to use the results which are known from an analysis of the direct potential layers
in application to the inclined double layer. In particular, if we assume $\epsilon_a = $ const

for the roughest estimates, then it is easy to see from (2.18a) that the thickness of the inclined layer along the axis η is equal (with not very small α);

$$ D_\eta = \frac{\Delta \Phi}{|\partial \Phi / \partial \eta|} = D_0 \left(1 + \varepsilon_a \alpha^2 \right)^{1/2} \simeq D_0 \frac{c}{v_a} \alpha . \tag{2.19} $$

from which it directly follows that the thickness of the layer in the direction Z equals

$$ D = D_\eta \cos\theta \simeq D_\eta \cdot \frac{1}{\alpha} = D_0 \sqrt{\varepsilon_a} = D_0 \frac{c}{v_a} \tag{2.20} $$

The quantity D_0 comprises, as is known [57], several Debye lengths: $D_0 = a\lambda_i$. By substituting the quantity D_0 in expression (2.20), we find that the thickness of the inclined double layer $D \approx a\rho_i$, i.e., comprises several Larmor radii of ions. Thus, the hypothesis made above that the thickness of the double layer is great as compared to the Larmor radii of ions is apparently fulfilled.

However, the numerical estimates presented above for the parameters of the inclined double layer are very approximate. The fact is that in these calculations we assumed that the quantity ε_a was constant, while it significantly changes in a direction perpendicular to the layer. In this respect, we will examine the parameters of the layer and conditions for its existence in more detail.

We will turn to the simplified hydrodynamic model examined in survey [50] in which temperature of the bundles of fast ions and fast electrons is assumed to be /49 negligible ($T_e = T_i = 0$). In this case the distribution of flux particles by longitudinal velocity is described by the delta-function, and equality (2.4a) is reduced to

$$ F_{i,e} = n_{i,e} . \tag{2.21} $$

By substituting the quantity $F_{i,e}$ from (2.21) into equality (2.11), we obtain the concentration of ions in the form

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$$n_i = n_{i1}\left(1 + \frac{e}{m_i\omega_{\theta_i}^2}a^2\phi''\right)\left(\frac{2W_{i1}}{m_i}\right)^{1/2}\left[\frac{2W_{i1}}{m_i} + \frac{2e}{m_i}(\phi_1 - \phi) - \frac{c^2a^2(\phi')^2}{B^2}\right]^{-1/2},$$ (2.22)

where, as previously $u_e/\phi e = \phi$ and $W_{i1} = \frac{1}{2}m_i u_{i1}^2$

The concentration of electrons, whose velocity of inertia drift is negligible, evidently equals

$$n_e = n_{e0}\left(1 + \frac{e\phi}{W_{e0}}\right)^{-1/2},$$ (2.23)

where

$$W_{e0} = \frac{1}{2}m_e u_{e0}^2$$

By substituting the equalities (2.22) and (2.23) in Poisson's equation (2.3) we can obtain an expression which determines the distribution of the potential ϕ in the layer. However, the equation thus obtained is still very complicated and is diffi-cult to analyze. In this respect, Swift [52] simplified the task even more, after assuming that the electrostatic charges are mainly concentrated near the boundaries of the layer, while inside of it, a condition is fulfilled that is only correct in that case where the layer thickness is much greater than the Debye radius, and there-fore is inapplicable in relation to the ion-sound layer examined in the previous section.

After equating expressions (2.22) and (2.23), we find:

$$\phi'' = \frac{m_i\omega_{\theta_i}^2}{ea^2}\left\{1 + \frac{n_{i1}}{n_{e0}}\left(\frac{W_{i1}}{W_{e0}}\right)^{1/2}\frac{(W_{e0} + e\phi)^{1/2}}{[W_{i1} + e(\phi_1 - \phi) - \frac{a^2e^2}{2m_i\omega_{\theta_i}^2}(\phi')^2]}\right\}$$ (2.24)

or in the dimensionless form:

$$\phi'' = 1 - \frac{n_{i1}}{n_{e0}}\left(\frac{W_{i1}}{W_{e0}}\right)^{1/2}\left\{\frac{\phi + W_{e0}}{1 + W_{i1} - \phi - \frac{1}{2}(\phi')^2}\right\}^{1/2}.$$ (2.24a)

Here ϕ is changed in units ϕ_1, W_{E0} and W_{i1} in units $e\phi_1$ and distances in the units $(a^2\phi_1/m_i\omega_{\theta_i}^2)^{1/2} = 1/\sqrt{2}\, a\rho_1$, where ρ_1—Larmor radius of the ion with energy $e\phi_1$.

Equation (2.24a) is numerically integrated with assigned values W_{eo} and W_{i1} and fixed boundary conditions $\phi(o) = 0$ with different values of (n_{i1}/n_{eo}). As a solution, the solution is selected in which with $\phi = 1$, $\phi^1 = 0$. The results of this integration with $W_{eo} = 0.1$ and $W_{i1} = 0.001$ are presented in Figure 2.2. On the x-axis the quantity $\eta/\Delta = \eta \cos\theta$, is plotted, i.e., the distance (in units selected above) in a direction perpendicular to the layer. We note that the quantity of the ratio (n_{i1}/n_{eo}) in this case was equal to 0.6, and not 0.1, as follows from the condition of Langmuir [50]. This indicates the implicit approximateness of the calculations. The thickness of the layer, as is apparent from the figure, was equal to the two units indicated above, i.e., about $1.5 \rho_i$. This agrees fairly well with the rough estimate presented above for this quantity.

In all the calculations presented above, the temperature of the bundle of ions and the bundle of electrons was assumed to be negligible. As shown by Swift [54], consideration for the final temperature of the particles in the bundle does not change the main conclusion regarding the existence of solution (2.24a) for the random value ϕ_1. The effect of the final temperature of the bundle is mainly reduced to increase in the thickness of the layer. This only improves the conditions of applicability of the discussed model based on the assumption that the thickness of the layer is much greater than the Larmor radius of energetic ions. The effect of temperature of energetic ions on the width of the inclined layer can be illus- /51 trated by Figure 2.3 [52]. It shows the distribution of potential in the layer in a model which contains a bundle of cold electrons above the layer and ions of two types: cold and hot, below the layer. As is apparent from the figure, increase in temperature of hot ions actually results in considerable (all the way to $\eta/\alpha = 10$) increase in the thickness of the layer.

The conditions for existence of the inclined double layer in the works of Swift, essentially have not been studied. The fact is that in the model for cold bundles and cold plasma described above, tnese conditions are essentially reduced to the Langmuir condition:

$$n_{eo}/n_{io} = (W_{io}/W_{eo})^{1/2}$$

(2.25)

In the hot plasma, the conditions for existence of the inclined double layer are reduced to the inequalities [50]

$$u_{ii} > \sqrt{T_{ei}/m_i}, \quad u_{eo} > \sqrt{T_{eo}/m_e}.$$

(2.26)

The process of formation of the inclined double layer and conditions for its development have also not been studied in detail at all. Nevertheless, certain qualitative considerations do exist on this account. Thus, Swift, in the already cited publication [52] examined the conditions for build-up of the cyclotron (n = u) instability (from which, the inclined double layer most likely develops), and showed that the indicated instability develops in that case where the relative velocity of the electrons exceeds the threshold value

$$u_e = a(T_e/m_i)^{1/2}$$

(2.27)

where a--factor on the order of a unit, which is below the threshold of development of ion-sound instability [19].

If the plasma contains ions of two types, cold and hot, then the threshold value of velocity for the bundle of electrons is equal to [52]:

$$u_e = \left(\frac{T_e}{m_e}\right)^{1/2} \left(\frac{T_e}{T_{ih}}\right)^{1/2} \frac{n_{ih}}{[n_o(n_o - n_{ih})]^{1/2}}$$

(2.28)

where T_{ih} and n_{ih}--respectively the temperature and concentration of hot ions, and /53 n_o--concentration of electrons. It is apparent from expression (2.28) that if the density of cold ions is low (so that $n_{ih} \approx n_o$), the threshold of development of tne
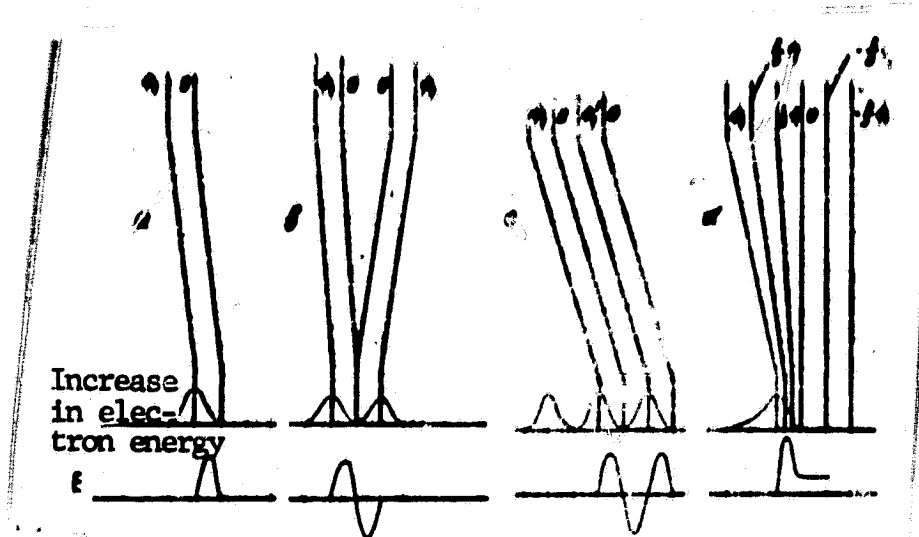
Figure 2.3. Distribution of Potential
Transverse to Inclined
Double Layer in Case of Hot
Plasma

examined instability drastically increases. Thus, the presence of relatively cold ions is a necessary condition for development of ion-cyclotron instability, and correspondingly, the appearance of inclined double layers.

In the model examined above for the inclined double layer, as in the model of a direct double layer, it was assumed that both above the layer and below it, the plasma is uniform. In the real magnetosphere, this is not so of course. In particular, in the exosphere, plasma density changes approximately according to the exponential law with a characteristic scale of altitudes $h = T/m_{ig}$. In this respect Swift [52] assumes that the area of possible existence of the inclined layer stretches along the force line of the geomagnetic field to a distance on the order of H. Below and above this region the force lines of the magnetic field can be considered equivalent. Possible configuration of the equipotentials is presented in Figure 2.4 [52]. This same figure shows the distribution (along the meridian) of the energy of electrons accelerated in the layer, as well as intensity of the electrical field at the level of the ionosphere.

Figure 2.4. Possible Configurations of Equivalent Elec-
tric Field in External, Ionosphere and Distri-
butions of Energy of Spilling Electrons Corres-
ponding to it and Horizontal Electric Field in
the Ionosphere

Figure 2.4a corresponds to the simplest situation described above for the
single inclined layer which is limited in altitude. As is apparent from the figure,
the field E in the ionosphere reaches the maximum in the center of the layer.

The energy of the accelerated electrons also has a characteristic latitudinal
profile with maximum near the "lower" boundary of the layer. In this case, the   /54
characteristic scale of the picture in the meridianal direction, as we have seen,
is 1 - 2 Larmor   radii of energetic ions, that is, 1 - 2 km at the level of the
ionosphere.

Figure 2.4b presents two layers adjacent to each other in which the horizontal
component of the electric field is directed to opposite sides.

Figure 2.4c corresponds to a multiplet structure of double layers and the
regions of precipitation of   accelerated particles which correspond to them.

Finally, Figure 2.4d corresponds to the case where outside the examined layer there is an electric field with layer not associated and directed strictly perpen-dicularly in relation to the magnetic field, so that the force lines of the latter are equipotential.

The limitedness of extension of the area for particle acceleration in altitude, and respectively, along the meridian results in an interesting effect which is acces-sible to the ground observer. The fact is, as we have already stated, in the case of ions of an inclined double layer which are fixed in relation to the ground observer, they must move in relation to the ions upwards and downwards, and in relation to the observer to the north or to the south with a velocity determined by the Langmuir condition (2.25). This condition requires that the layer move along the force line with a velocity on the order of $u_{i*} = u_{eo} \frac{n_i}{n_i} \sqrt{\frac{m_e}{m_i}}$. On the condition of $n_e/n_i \approx 1$ and $u_{eo} = 3 \times 10^9$ cm/s ($W_e = 2.5$ keV) this yields $u_i = 2 \times 10^7$ cm/s. In this case, the velocity of movement of the layer (and correspondingly, the arcs of the aurora borealis) along the meridian equals u_i/α , and with $\alpha \approx 10^3$, this velocity is on the order of 20 m/s. This does not contradict the experimental data.

These are the parameters of the inclined double layers which are suggested /55 based on the theoretical examination. Under laboratory conditions, these layers have not been studied as yet, so for experimental verification of the obtained laws, we will turn to the data of direct observations for these layers in the magneto-spheric plasma.

2.2. DOUBLE POTENTIAL LAYERS IN THE MAGNETOSPHERE; TWO-DIMENSIONAL MODEL OF THE LAYER

Figure 2.5 (lower panel of the figure) presents typical behavior of three components of the quasistationary electric field when the satellite S 3 - 3 moving

at an altitude about 2000 km intersects the zone of the aurorae boreales [58].

From the data presented in the figure it is evident that the external ionosphere at invariant latitudes 68° - 72°, extremely inhomogeneous electric fields are observed with intensity in a direction transverse to the magnetic field $(E_\perp)$ up to 300 mV/m and in a longitudinal $(E_\parallel)$ up to 30 mV/m, In this case, judging from the flight time of the satellite through the region of the strong electric field, the boundaries of this region along the satellite trajectory comprise no more than several kilometers.
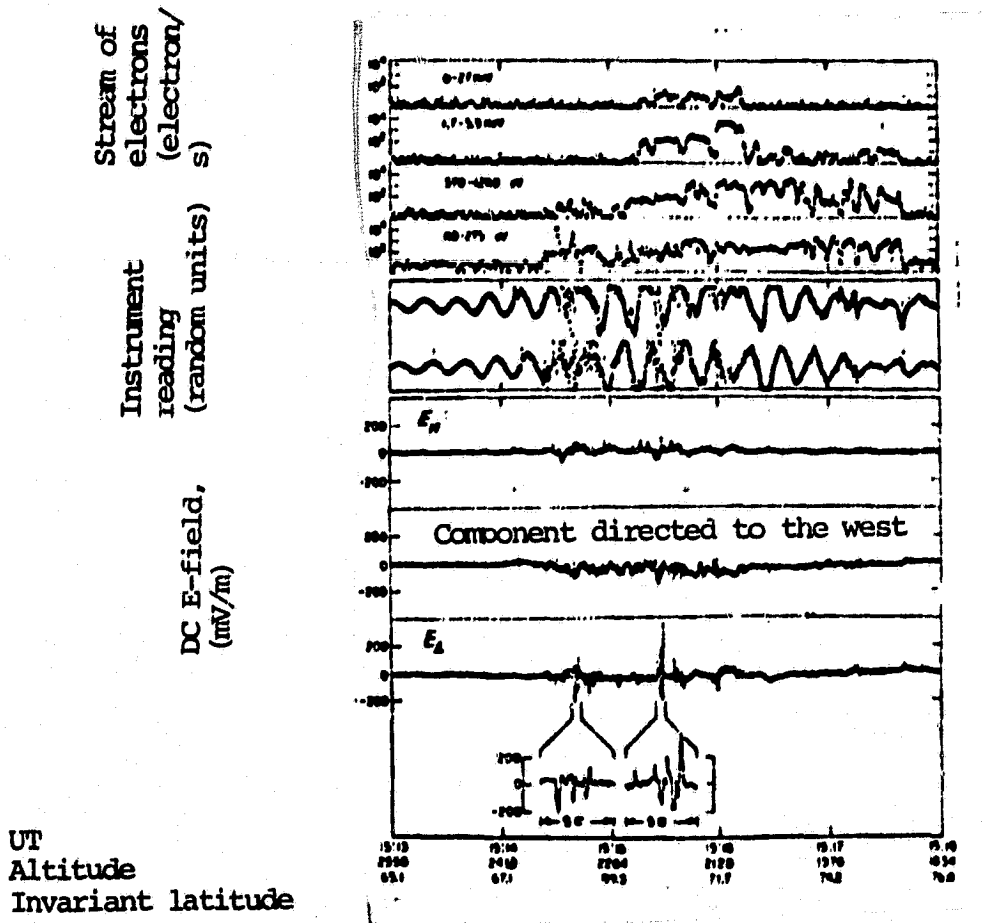
/56

Figure 2.5. Electric Field and Observed Stream of Electrons During Typical Passage of Satellite S 3 - 3 Through Region with Paired Electric Shock Waves. Below is a more detailed time scan in a scale corresponding to the width of the arc.

48

Such small dimensions of the region for the existence of the field, like its high intensity, make it possible to identify this region with the double electrostatic layer. The quantities indicated above for the longitudinal and transverse components of the electric field indicate that this layer is inclined in relation to the magnetic field, and apparently correspond to the model described above for the inclined ion-cyclotron layer. This hypothesis is confirmed by the circumstance that the examined area, as can be seen from the data presented in Figure 2.5, was obtained in a broader region of intensive electrostatic turbulence. This agrees quite completely with the theoretical model of the layer suggested by Swift.

Detailed analysis of the extensive material obtained on satellite S 3 - 3, permitted the authors of [58, 59] to draw the following conclusions regarding the parameters of the double layers in the magnetosphere:

1. The double layers were found in the entire range of altitudes of action of satellite S 3 - 3 from 1000 to 8000 km, and the probability of appearance of tne layer increases with a rise in altitude.

2. The dimensions of the double layer along the meridian are (in calculation for the 100 km level) from 200 m to 10 km.

3. The intensity of the transverse component of the electrical field in the layer reaches a quantity on the order of several tens of volts per meter, all the way to 1 V/m. Similar results were obtained previously from the data of launchings of barium clouds and streams at altitudes of several thousands of kilometers [60, 61]. The longitudinal component of the field averages an order weaker.

4. The total difference in potentials transverse to the layer is several (all the way to 10 or even more) kilovolts.

5. Double layers are observed in the region of relatively intensive longitudinal currents ($j \cong (1 - 10) \ 10^{-10}$ A/cm^2), and are accompanied by plasma turbulence with

amplitude of the field fluctuations to 50 mV/m.

It is easy to see that the observed characteristics of the double layers in the magnetosphere on the whole are fairly close to the properties predicted by theory.

The difference in potentials existing transverse to the layer in this case is quite sufficient to accelerate the auroral electrons to the observed energies. This circumstance makes it possible to hypothesize that the double layers observed in the magnetosphere must be linked to bright discrete forms of aurorae boreales. And in fact, direct comparison of specific cases of observed double layers with the aurorae boreales convincingly confirm this hypothesis [58]. Study of the link between the double layers in the magnetosphere and the aurorae boreales can be significantly expanded and deepened, as applied to precipitations of electrons of the "inverted" v type.

Precipitations of this type were reported for the first time by Frank and Ackerson [62, 63]. The essence of this phenomenon is the following. Figure 2.6 [64] shows the distribution (along the rocket trajectory) of luminescence intensity, density of the stream of energy of the precipitating electrons and their energy (in the region of the spectral peak). As is apparent from the figure, with intersection of the arc of luminescence, the energy of the electrons increases, reaches a certain maximum (in this case over 10 keV), and then again diminishes, outlining on the graph a figure which in shape is similar to an "inverted" letter v. In order to explain these features in the distribution of intensity of the stream and energy of the precipitating particles, Garnett [65] suggested that precipitations of the examined type are associated with the existence in the upper ionosphere of an electric field whose characteristic distribution in space is described by a system
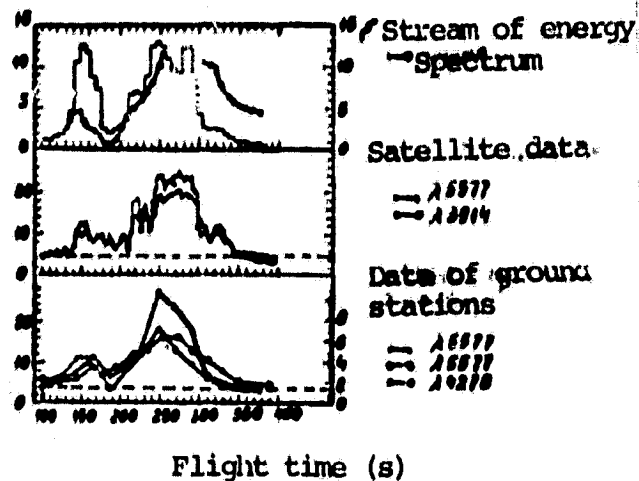
Figure 2.6.   Distribution of Intensity
of Precipitating Electrons
and Brightness of Luminescence
According to Data of Satellite
and Ground Stations

of equipotentials of the type presented in Figure 2.7. In fact, it is easy to see

that in this case the particles which move along the central axis of the figure

intersect the greatest number of equipotentials, and correspondingly, the maximum

acceleration results. The energy of the precipitating particles diminishes the

farther from the axis of the figure.

We note that somewhat earlier, an analogous model of spatial distribution of

the electric field in the auroral ionosphere and the equipotentials describing it

was suggested by Carlquist and Bostrom [66]. The model they suggested was based on

results of observing the movement of inhomogeneities of aurorae boreales rays in the

auroral arcs [66, 67]. These observations which showed that inhomogeneities ob-

served to the south and to the north of the central axis of the arc move along the

arc to different sides with a velocity required for the existence of the electric

field $E_\perp \sim 0.5$ V/m (in conversion for the level of the ionosphere). Since these

intensive fields were never observed directly in the ionosphere, the authors [66]

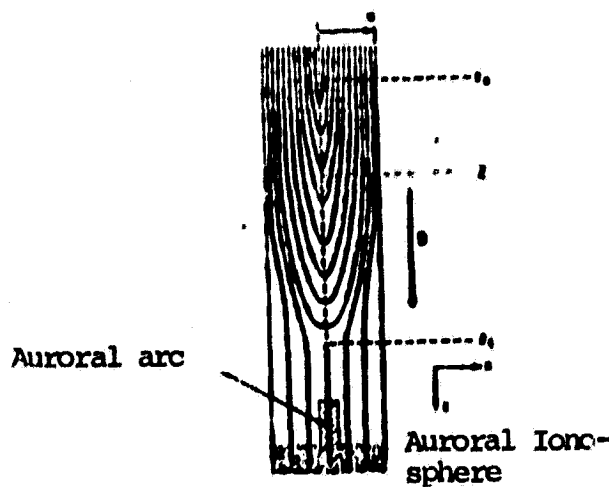also suggested a model similar to that depicted in Figure 2.7.

Figure 2.7.   Distribution of Equipotentials
of Electric Field in Model "V-
Shock Wave."

Direct observations of the electric field in the magnetosphere that were made

on the satellite S 3 - 3 show that the double layers are encountered most often in

pairs or even series, and they are such that the electric field ($E_\perp$) in the adjacent

layers is directed to different sides [58, 68].

Theoretical substantiation of the indicated distribution of electrostatic poten-

tial was given by Swift [52, 54]. The model he suggested is a development of the

model examined above for a planar inclined double layer. It consists of the follo-

wing. Assume that outside the region $a \leq x \leq a$, the longitudinal electric field is

missing. Within the indicated interval, distribution of the potential at level z =

z_0 and at level z = z_1 is assigned (similarly to Figure 2.1):

$$\phi(x)\Big|_{z=z_0} = \phi_0(x)$$

$$\phi(x)\Big|_{z=z_1} = \phi_1(x)$$

(2.29)

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On the central axis of the figure, because of its suggested axisymmetry

$$\frac{\partial \phi}{\partial x}\Big|_{x=0} = 0 \ . \tag{2.29a}$$

Distribution of the potential in the layer, i.e., in the region $|x| \leq a$ and $z_0 < z < z_1$, is provided by Poisson's equation (2.16). Since in the two-dimensional case we have examined, solution to this equation is excessively complicated, we will introduce the following simplifications. Since, as follows from experiment, $E_\parallel \ll E_\perp$, we assume that everywhere with the exception of the axis, the layer (X = 0) $\varepsilon_a \frac{\partial^2}{\partial x^2} \gg \frac{\partial^2}{\partial z^2}$ and $\frac{\partial \varepsilon_a}{\partial x} \frac{\partial}{\partial x} \gg \frac{\partial^2}{\partial z^2}$.

Then equation (2.16) adopts the appearance

$$\varepsilon_a \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2} \frac{\partial \varepsilon_a}{\partial x} \frac{\partial \phi}{\partial x} = -4\pi e n_e^* \left(\frac{n_i^*}{n_e^*} - 1 \right). \tag{2.30}$$

Concentration of particles included in the right side of equation (2.30) is defined by the unknown distribution of potential, and by the initial energy W_{eo} of fast electrons above the layer and W_{i1} of fast ions below the layer, as well as by the distribution along the axis x of their concentration n_{eo} and n_{i1}. However, random assignment of the last two quantities is impossible, since the correlation between them determines the size of the jump $\Delta \phi(\cdot) = \phi_1 - \phi_{0}$, which is already assigned. In this respect, Swift suggests the following procedure for solving equation (2.30). We select a certain quantity $\phi(o) = \phi_2(o)$ where ϕ_2 is slightly less than $\phi_1(0)$, and we numerically integrate in equation (2.30). Integration is done until the equality $\psi_2(x_2) = \phi_1(x_1)$ is fulfilled at a certain point $x_2 < a$. However, in order for the density of the charge to remain finite everywhere, we additionally require that $(\partial \phi/\partial x)_{x_1} = (\partial \phi/\partial x)_{x_1}$. This condition can be satisfied only by the appropriate selection of the size of the ratio of primary density of fast ions in the region $z \leq z_1$ to the density of fast electrons in the region $z \leq z_0$ at the point

$x_{\perp}\left(R(r_{\perp}) \cdot \frac{n_{ij}}{n_{eo}}\right).$ After selecting the quantity $R(x_2)$ in the necessary manner, we

again integrate equation (2.30) on the hypothesis of linear relationship $R(x)$ in

the interval $0 \le x \le x_2$. We then select a new value for the potential on the axis

of the figure $\phi_3(0) < \phi_2(0)$ and equation (2.30) is integrated with the same boundary

conditions to point x_3 at which $\psi_3(x_3) = \phi_1(x_3)$.   A new value $R(x_3)$ is selected.

This process is repeated many times until a sufficient number of profiles of $\phi(x)$

is obtained which correspond to different values of $\Phi(0)$, i.e., integrating the

central axis at different altitudes. The results of this calculation are presented

in Figure 2.8a [54]. The isolines of Φ which are thus obtained are presented in

Figure 2.8b. The lower part of this figure shows the distribution along the x-axis

of energy of electrons accelerated in the layer. It is apparent from the figure

that both the configuration of the isolines Φ, and the distribution of energy of the

precipitating particles obtained in the model qualitatively coincide with similar

characteristics of precipitations of the "inverted" v type. The analysis made by

Swift for the possible existence of the v-type double layer also contains, although

in an implicit form, the answer to the question of what governs the characteristic

shape of the layer. In fact, as we have just seen, a definite distribution of the

quantity $R(x) \cdot \frac{n_{io}(x)}{n_{eo}(x)}$ transverse to the layer is necessary for the existence

of a layer of the indicated shape. We assume that the appearance of a double layer

is governed by the intrusion of energetic electrons into a uniform (i.e., n_{i1} = const)

ionosphere. Then both the actual fact of existence of a v-type layer, and its posi-

tion in space, and apparently also the dimensions are determined by the distribution

$n_{eo}(x)$. Thus, depending on the distribution of intensity of the bundle of primary

electrons, both a v-shaped layer of random width (we recall, that in the case of a

uniform bundle in uniform plasma, a flat layer is formed with width on the order of

several Larmor radii), and a simple layer of a certain configuration can appear
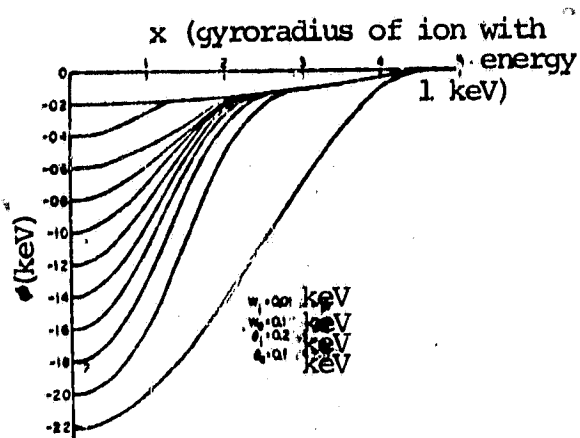
(see Figure 2.4).

Figure 2.8(a) Calculated Distribution
of Electric Potential in
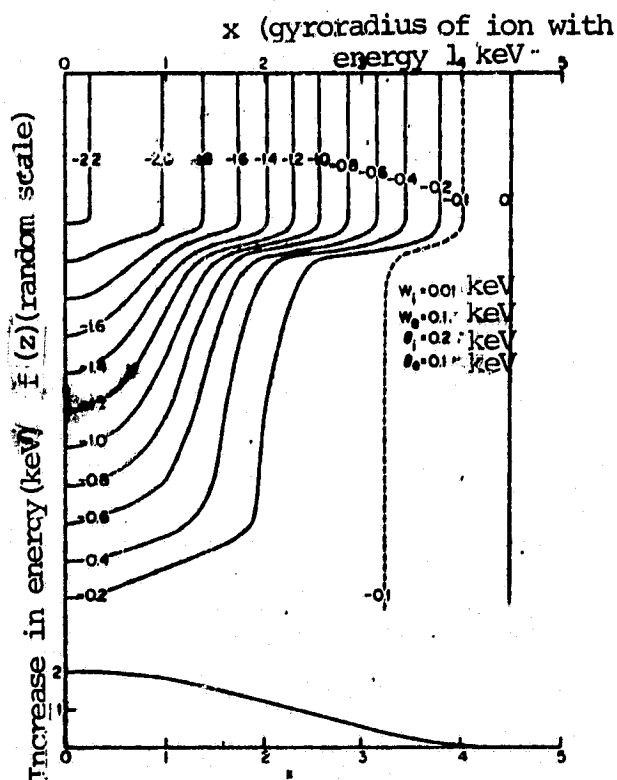"V-Shock" Wave in Parametric
Space

Figure 2.8(b) Spatial Distribution of
Potential

55

Thus, the experimental data examined above make it possible to assert that double potential layers are just as widespread a phenomenon as anomalous resistivity, and the difference in potentials existing in them is commensurate with the drop in voltage in the region of anomalous resistivity and is sufficient to accelerate particles to typical auroral energies.

It is thus apparently difficult at this time to state which of the discussed phenomena can be viewed as the main mechanism for accelerating auroral electrons along the force lines of the magnetic field. It should be noted however, that judging from the fact that double layers in all cases where they were observed, were submerged into strongly turbulent plasma, as well as from the fact that their transverse dimensions are 1 - 2 orders smaller than the typical width of the zone of precipitation of energetic electrons, one can assume that on the whole they play a secondary role in the energetics of auroral phenomena, being responsible more likely for the relatively narrow and short-lived, although clear discrete forms of aurorae boreales. Thus, double layers in the magnetospheric plasma are apparently a certain secondary effect or partial, although they may be a extreme manifestation of electrostatic turbulence of plasma in the presence of strong longitudinal currents.

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